FIBRE SPACES WITH TOTALLY DISCONNECTED FIBRES

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By *fibre space,* we shall mean fibre space in the sense of *Hu* [3]. In this paper, we determine necessary and sufficient conditions that a fibre space with totally disconnected fibres, and arcwise connected base space be an E-F fibre bundle [6; 18]. The result is then applied (Theorem 4) to fibre bundles to obtain a "minimal" bundle equivalent to the given bundle. Applications to locally compact groups are given in the final section.

1. **Fiberable spaces.** A system $\mathfrak{F} = [X, B, \pi]$, where X and B are topological spaces, and π is an open, continuous mapping of X on B, will be called a *fiberable space.* X is called the *total space,* B the *base space,* π the *projection,* and $\pi^{-1}(b)$, $b \in B$, the *fibre over b.*

We shall say that a fiberable space $\mathfrak{F}$ with homeomorphic fibres is *tied* if there exists a collection $[Y, G, H_b]$, where Y is a homeomorph of $\pi^{-1}(b)$, $b \in B$, G is a group (not topologized) of homeomorphisms of Y onto itself and H_b, $b \in B$, is a set of homeomorphisms of Y onto $\pi^{-1}(b)$ satisfying

$$h_1, h_2 \in H_b \quad \text{implies} \quad h_2^{-1}h_1 \in G, \tag{1}$$

$$h \in H_b, \; g \in G \quad \text{implies} \quad hg \in H_b. \tag{2}$$

An E-F *bundle* [6; 18] is a collection $\mathfrak{B} = [X, B, Y, \pi, \psi_b, V_b, H_b, G]$ where $[X, B, \pi]$ is a fiberable space which is tied by $[Y, G, H_b]$, and for each $b \in B$, there exists a neighborhood, called a *coordinate neighborhood,* V_b of b, and a mapping $\psi_b : V_b \times Y \to \pi^{-1}(V_b)$, called a *coordinate function,* which is an onto homeomorphism with the property that $\pi\psi_b(a, y) = a$, $a \in V_b$. It is also required that ψ_b, when restricted to $b \times Y$, be an element of H_b.

We recall that a *fibre space* (in the sense of Hu [3]) is a collection $\mathfrak{F} = [X, B, \pi, \phi_U, \Omega]$, where $[X, B, \pi]$ is a fiberable space, Ω is a collection of open sets U covering B, and where, for each U, ϕ_U is a continuous function, called a *slicing function,* mapping $U \times \pi^{-1}(U)$ onto $\pi^{-1}(U)$ such that

$$\phi_U(\pi(x), x) = x \qquad (x \in \pi^{-1}(U)), \tag{3}$$

$$\pi\phi_U(b, x) = b, \qquad (b \in U, \; x \in \pi^{-1}(U)). \tag{4}$$

Consider now the E-F bundle $\mathfrak{B}$ as described above. Let $\psi_{b,a}(y) = \psi_b(a, y)$. Define

$$\phi_b : V_b \times \pi^{-1}(V_b) \to \pi^{-1}(V_b)$$

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