IRREDUCIBLY CONNECTED SPACES

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1. **Introduction.** A connex (connected space) M is said to be *irreducibly connected* about a subset A if M contains A, but no proper sub-connex of M contains A. For briefness, such a connex M will be called an i-connex about A. In this paper, a study is made of certain aspects of irreducible connectedness not previously considered, and certain questions not adequately dealt with by previous investigators are more fully answered.

Unless otherwise specified, the space under consideration will be a Hausdorff space. Two non-vacuous sets R and Q are said to be *separated* if $\bar{R} \cdot Q = 0 = R \cdot \bar{Q}$. If a set M is expressed as the sum of two such sets, this will be stated briefly as $M = R + Q$ (sep).

2. **The subsets about which a given connex is irreducibly connected.** · Since any connex M is irreducibly connected about itself, there is always at least one subset A about which M is an i-connex. The number of different subsets A depends upon the number of cut points of the space M.

2.1 Theorem. *If a connex M has exactly n cut points, where n is an integer, then there are exactly 2^n different subsets A about which M is an i-connex.*

Proof. Let K denote the set of cut points of M, and let D be any sub-collection of K.

If M is not an i-connex about $M - D$, some proper sub-connex R of M contains $M - D$. Let q be any point of $M - R$. Since q is a point of K, $M - q = P + Q$ (sep). Since R is connected, R is contained in one of these separated sets, say $R \subset P$. Then $Q + q$ is a non-degenerate connected subset of M, and $Q + q \subset M - R \subset K$. But then $Q + q$ consists of a finite number of points. This is a contradiction, since a non-degenerate connex must contain infinitely many points. Therefore M is an i-connex about $M - D$.

Since every subset A about which M is an i-connex must contain $M - K$, every subset A is of the form $M - D$. Since M has n cut points, there are exactly 2^n different choices for D.

2.2 Corollary. *If a connex M contains infinitely many cut points, then M is an i-connex about infinitely many different subsets, and conversely.*

The most characteristic property of the subsets about which a connex M is irreducibly connected is that each such subset contains all of the non-cut points of M. This was first pointed out by Gehman [1]. No comments have been

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