THE UNIVALENT ALGEBRAIC TRANSFORMATIONS OF THE PROJECTIVE PLANE

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1. **Introduction.** Let Δ be a field and let U be a universal field extension of Δ, that is, let U be an algebraically closed field extension of Δ whose degree of transcendency over Δ is infinite. Let P be the projective plane over U, that is, the set of all projective points whose coordinates are elements of U (see [2]).

We shall say that f *is a univalent algebraic transformation of* P *over* Δ, if, and only if,

(1) f is an algebraic variety in $P \times P$ over Δ,

(2) if s is a point in P and s is algebraic over Δ then there exist uniquely determined points t and u in P such that (s, t) and (u, s) are elements of f.

It follows readily that the statement (2) remains true if the condition that s is algebraic over Δ is dropped, and that in that case t and u have the same dimension over Δ as s. We shall denote t by $f(s)$ and u by $\operatorname{inv} f(s)$.

If f and g are univalent algebraic transformations of P over Δ then their composition $f \circ g$, defined by $(f \circ g)(s) = f(g(s))$ for all points s in P, is also a univalent algebraic transformation over Δ. Hence the set of all univalent algebraic transformations in P over Δ defines a group G with the composition as group operation. It is well known that G coincides with the group of projective (*i.e.*, homogeneous linear) transformations of P over Δ in case the characteristic of Δ is zero. This result is no longer true in case Δ has a characteristic p which is not zero. In the latter case G is considerably larger than the group of projective transformations. It is the purpose of this paper to construct a set of generators for G in that case.

We shall first assume that Δ is a perfect field with a characteristic p ($p \geq 2$). Let O_0, O_1, O_2, E be a projective basis of P which is rational over Δ, that is, for which each of the points O_0, O_1, O_2 and E has coordinates that are elements of Δ. Let O_0', O_1', O_2', E' be another projective basis which is rational over Δ. Let σ be a homogeneous polynomial in two variables x_1, x_2 and with coefficients in Δ. Let degree $\sigma = p^m$ and assume σ is not the p-th power of another polynomial in x_1, x_2 over Δ. Let e be an integer (positive, negative, or zero). Finally let S be the function on P onto P such that whenever $s = s_0O_0 + s_1O_1 + s_2O_2$ is a point in P then

$$S(s) = \{s_0^{p^m} + \sigma(s_1, s_2)\}^{p^{-e}} O_0' + s_1^{p^{m-e}} O_1' + s_2^{p^{m-e}} O_2' .$$

It can be readily verified that

(1) S is an element of G,

(2) if $m = 0$, then S maps each straight line in P onto a straight line in P;

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