THE DIFFERENTIALS OF CERTAIN FUNCTIONALS IN EXTERIOR BALLISTICS

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1. **Introduction.** In computing the range of a projectile one is faced with the problem of calculating or closely approximating the solution of a differential equation involving several parameters and functions which depend on the time and place of the firing of the projectile. A complete tabulation of solutions, covering all possible wind-structures, density distributions, *etc.*, is manifestly impracticable. But this difficulty has been resolved by a procedure which is feasible and (apart from rare exceptions) provides adequately accurate approximations to the desired quantities, such as range and time of flight. The procedure is to compute the trajectories under certain relatively simple "normal" conditions, and then to replace the intricate exact effect of a departure from "normal" conditions by a linear approximation, the "differential effect" of the departure.

To be specific, let us restrict our attention to the effect of wind on range. The "normal" condition, under which the trajectories are computed, is zero wind. The range is a functional of the wind. If the wind is supposed to be a continuous function of altitude alone, we are considering a real-valued functional on a space of continuous functions. In this space we can introduce a familiar norm, the **norm** of a continuous function $w(\cdot)$ being defined to be the least upper bound of its absolute value. Then the classical definition of a (first) differential can be applied; it is a linear functional of $w(\cdot)$ which differs from the change of range ΔX due to $w(\cdot)$ by a quantity whose ratio to norm $w(\cdot)$ approaches zero with norm $w(\cdot)$. But the existence of a differential in this sense has not heretofore been established. To the best of my knowledge, the only adequate mathematical discussion of differential effects is due to G. A. Bliss (see [1] and [2]). Bliss, however, does not use the norm mentioned above. Instead, he defines the norm of $w(\cdot)$ to be the greater of the least upper bound of $|w|$ and the least upper bound of the absolute value of its derivative with respect to altitude y; that is, he uses the norm of the Banach space C' instead of the norm of the space C. Now with certain disturbances this would be quite unobjectionable. For instance, the Coriolis force on a projectile is a disturbance whose value and rates of change with respect to position and velocity are computable, and the inclusion of the rates of change in the definition of the norm produces no annoyance. But the wind is an experimentally determined function and the experiment furnishes only the mean values between a finite number of successive altitudes. The experimental result may be expected to differ from the exact wind by an error whose norm in the space C (*i.e.*, maximum absolute value) is small; but we have no assurance that the rate of change of wind with

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