EXTENSIONS OF THE PONTRJAGIN DUALITY
I: INFINITE PRODUCTS

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1. **Introduction.** An as yet unsolved problem is to characterize the class of topological abelian groups for which the Pontrjagin Duality holds, that is, those groups which are the character groups of their character groups. As is well known, this class includes the class of locally compact (abelian) groups, but it is actually larger, as we shall see in this paper. A special case of the above general problem is: under what operations is this class closed? For example, is it closed under the operations of taking closed subgroups and factor groups modulo closed subgroups? Or under the operation of taking infinite products of groups? Or under the operations of taking inverse limits and direct limits (suitably defined)?

In the present paper, we answer the second of the above in the affirmative, that is, we show the class is closed under the operation of taking infinite products. As a matter of fact, we deal with two kinds of infinite products. One is the usual (full) product of a set of groups $\{G_\lambda\}$, which we denote by $\mathbf{P}G_\lambda$, with the usual Tychonoff topology. The other is the weak product—that is, the subset of $\mathbf{P}G_\lambda$ consisting of those points with only a finite number of coordinates different from zero—which we denote by $\mathbf{P}^w G_\lambda$, with a very fine topology, defined later (§3). Specifically, we prove the following duality theorem.

Let $\{G_\lambda\}$, $\{H_\lambda\}$ be two sets of groups such that for each λ, G_λ and H_λ are the character groups of each other. Then $\mathbf{P}G_\lambda$ and $\mathbf{P}^w H_\lambda$ are the character groups of each other.

It follows of course that the class of groups for which the Pontrjagin Duality holds is closed under the operations of taking either products or weak products. In particular, the products of locally compact groups have the Duality property, which has not been hitherto known. Since the infinite product of locally compact groups need not be locally compact, this incidentally shows that the class of groups with the Duality property is larger than the class of locally compact groups.

Using our topology for the weak product, it is possible to define the limit group of a direct system of topological groups in a natural way (see [1; II, 14] for the discrete case). We shall, in a succeeding paper, examine the question of whether the class of groups with the Duality property is closed under inverse and direct limits.

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