POWER SERIES DEVELOPMENTS FOR THE EQUATIONS OF A GENERAL ANALYTIC VARIETY IN HYPERSPACE

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1. Introduction. A covariantly determined reference frame for the definition of local point coordinates and associated power series developments for the equations of curves and surfaces are of fundamental importance in the projective differential geometry of ordinary space. The power series developments for a space curve express two non-homogeneous local coordinates of a point of the curve in terms of the third non-homogeneous coordinate; the power series development for a surface expresses one non-homogeneous local coordinate of a point of the surface in terms of the two remaining non-homogeneous coordinates. The usual process of deriving such developments is a tedious one which consists of determining auxiliary expansions for homogeneous coordinates in terms of curvilinear parameters, dividing each of these auxiliary expansions by one of them to determine expansions for non-homogeneous coordinates, and replacing the auxiliary parameters in these expansions by expressions calculated for them in terms of the same number of non-homogeneous coordinates.

In the present paper the much more general problem of determining power series expansions in local non-homogeneous coordinates for the equations of an m-dimensional variety V_m in an n-dimensional linear space S_n is solved by a simple direct method which eliminates the use of auxiliary power series in curvilinear coordinates. To the author's knowledge no such power series developments have been heretofore derived for the equations of a V_m in a linear space S_n. This is due, no doubt, to the great complications which arise when one seeks to extend the usual process to this case of $n - m$ developments in terms of m independent non-homogeneous coordinates. These power series developments are, however, easily derived by the method of the present paper. Recursion relations are readily obtained which express the coefficients of the terms of degree k in terms of those of degrees less than k. Special cases of these relations are applied to ordinary three-dimensional projective space to derive (1) the canonical expansions for the equations of a curve referred to the Laguerre-Forsyth canonical form of the differential equation, and (2) the canonical expansion for the equation of a surface due to Lane and Clement [2; 129–132].

The method employed in the present paper may be briefly described as follows. A reference frame for local coordinates of a point X is chosen whose vertices $x_0, x_1, \cdots, x_n$ vary as the vertex x_0 moves over the variety V_m. Let X denote a neighboring point of x_0, lying on V_m. The conditions (2.8) that the point X be fixed as x_0 moves along a parametric u^ϵ curve are determined.

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