FOURIER-WIENER TRANSFORMS OF FUNCTIONALS BELONGING TO L_2 OVER THE SPACE C

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1. Introduction. In this note we shall show that every real or complex-valued functional $F(x)$ which belongs to $L_2(C)$,

$$(1.1) \qquad \int_C^w | F(x) |^2 d_w x < \infty ,$$

has a Fourier-Wiener transform $G(x)$ which also belongs to $L_2(C)$ and whose transform is $F(-x)$. Moreover, we shall show that F and G satisfy Plancherel's relation in the form

$$(1.2) \qquad \int_C^w | F(x) |^2 d_w x = \int_C^w | G(y) |^2 d_w y.$$

We first define the Fourier-Wiener transform for functionals belonging to a subclass E_m of $L_2(C)$ and then we show that these functionals are dense in $L_2(C)$. The class E_m is a class previously considered [1]; the definition of the Fourier-Wiener transform used here will differ slightly from that used in [1] and [2].

In showing that the functionals of E_m are dense in $L_2(C)$ we will use the Fourier-Hermite development considered in [3]. The desirability of having Plancherel's relation in the form (1.2) rather than in the form given earlier [1; (1.2)] led us to adopt the modified definition of the transform which we use here.

Throughout this note C will denote the space of real-valued continuous functions defined on $0 \leq t \leq 1$ which vanish at $t = 0$, and the measure used on C shall be that defined by Wiener (see [6], where references to his earlier papers are also given). The space $L_2(C)$ will consist of all real or complex-valued W-measurable functionals $F(x)$ satisfying (1.1) In the process of proving the main result on transforms of functionals of $L_2(C)$, we will use functionals defined throughout the space K of complex-valued continuous functions defined in $0 \leq t \leq 1$ which vanish at $t = 0$; but we emphasize that for the result itself the functionals need not be defined over K but only almost everywhere over C.

2. **Fourier-Wiener transforms of functionals of class E_m.** As in [1] we consider the class E_m of functionals $F(x)$ which are defined throughout the space K and which are mean continuous, "entire", and of mean exponential type. That is, E_m is the class of functionals satisfying the following three conditions:

$$1^\circ \qquad \lim_{n\to\infty} F[x^{(n)}] = F[x]$$

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