DIVISOR FUNCTIONS OF POLYNOMIALS IN A GALOIS FIELD

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1. **Introduction.** Suppose n is a positive integer and p a positive prime. Then $GF(p^n)$ denotes the Galois field of order p^n and $GF[p^n, x]$ the ring of polynomials with coefficients in $GF(p^n)$. A primary polynomial of $GF[p^n, x]$ is a polynomial whose leading coefficient is the unit element of the field. In this paper all polynomials will be assumed primary and will be denoted by capital italic letters, while corresponding small italics will indicate the degree of the polynomials.

Let M be a polynomial of degree $2k$ ($k \geq 0$). Then we define the following divisor functions:

$$(1.1) \qquad \delta_z(M) = \begin{cases} \displaystyle\sum_{Z \mid M}^{\deg Z = z} 1 & (z \geq 0) \\ 0 & (z < 0), \end{cases}$$

$$(1.2) \qquad \gamma_z(M) = \delta_z(M) - \delta_{z-1}(M),$$

and

$$(1.3) \qquad \rho_s(M, \mu) = p^{2kns} \sum_{z=0}^{k} \mu^z p^{-nzs} \gamma_z(M),$$

where μ is a parameter and s is an arbitrary complex number. In particular, when $\mu = 1$, we write

$$(1.4) \qquad \rho_s(M) = \rho_s(M, 1).$$

We note in passing that the δ_z -function (1.1) is simply the τ^z function of [1], while $\rho_s(M)$ in (1.4) is the ρ-function of [2] in a new form involving γ_z.

We now define functions which generalize those just given. Let e be a fixed positive integer with $\deg M = 2ek$. Put

$$(1.5) \qquad \delta_z^e(M) = \begin{cases} \displaystyle\sum_{Z^e \mid M}^{\deg Z = z} 1 & (z \geq 0) \\ 0 & (z < 0), \end{cases}$$

$$(1.6) \qquad \gamma_z^e(M) = \delta_z^e(M) - p^{n(1-e)} \delta_{z-1}^e(M),$$

and

$$(1.7) \qquad \rho_s^e(M, \mu) = p^{kn\{e(s-1)+s+1\}} \sum_{z=0}^{k} \mu^z p^{nz(e-s-1)} \gamma_z^e(M);$$

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