# THE THEORY OF CONTACT OF CURVES IN A PROJECTIVE SPACE OF $N$ DIMENSIONS

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1. **Introduction.** The projective theory of contact of two plane curves has been enriched by E. Bompiani [1], who considered, as a result of the cross ratio appearing in the classical method of C. Segre, a certain line $r_0$ projectively connected with the point of contact. In the case where the two curves have contact of the first order the line $r_0$ can be defined as the limiting position of the line joining two corresponding points of the curves such that the tangents drawn for each curve meet in a point on the common tangent of the curves. This elegant interpretation of $r_0$ due to S. C. Chang [2] however cannot be generalized to the case where the order of contact is greater than 1, for the limiting position in this case always coincides with the common tangent.

In order to generalize the classical invariant of Segre for two plane curves, B. Segre [3] has shown that there are $n - 1$ projective invariants of contact for two curves in an $n$-dimensional space if they have at their common point all the osculating spaces in common. This remarkable generalization can further be generalized to the case where the two curves have at their common point the same osculating linear spaces of dimensions $1, 2, \cdots, k$, where $2 \leq k \leq n - 1$. (See [5].)

The object of the present paper is to derive a certain system of linear spaces associated with the common point of contact of the curves. This may be seen on the one hand as a generalization of Bompiani line $r_0$ to the general case and on the other hand as a supplement to the previous research of the present author.

2. **The invariants of contact.** Suppose that two curves $C$ and $\Gamma$ in a projective space $S_n$ of $n$ dimensions have at their common point $P$ the same osculating spaces of dimensions $1, 2, \cdots, k$, where $2 \leq k \leq n - 1$. Then the expansions of these curves in the neighborhood of $P$ referred to a suitable system of reference take the form, namely,

$$(1)\quad \begin{cases} Z^i = \dfrac{1}{i!} \displaystyle\sum_{j=0}^{\infty} b^i_{i+j} (Z^1)^{i+j} & (i = 2, \cdots, k), \\ Z^l = \dfrac{1}{l!} \displaystyle\sum_{j=0}^{\infty} b^l_{k+1+j} (Z^1)^{k+1+j} & (l = k+1, \cdots, n) \end{cases}$$

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