INTEGRAL TRANSFORMS

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The purpose of this paper is to apply the fundamental methods of Carleman and Stone to analyze the structure of certain well-known transformations of the form

$$f(s) = \int_{-\infty}^{\infty} H(s, t)\varphi(t)\, dt, \qquad \varphi(t) \in L^2(-\infty, \infty).$$

Of particular importance are the following kernels

(1)	$\frac{1}{2}e^{-\vert s-t\vert}$	(Picard),
(2)	$\pi^{-\frac{1}{2}}e^{-(s-t)^2}$	(Weierstrass),
(3)	$\pi^{-1}[1 + (s - t)^2]^{-1}$	(Poisson),
(4)	$(2\pi)^{-1} \operatorname{sech} \frac{1}{2}(s - t)$	(Stieltjes),
(5)	$\sin^2 (s - t)/\pi(s - t)^2$	(Fejér),
(6)	$\sin (s - t)/\pi(s - t)$	(Dirichlet),
(7)	$\pi^{-\frac{1}{2}} \exp\left(-e^{s+t} + \frac{s + t}{2}\right)$	(Laplace).

It will be observed that these special kernels are all of the form $H_1(\vert s - t \vert) + H_2(s + t)$, a kernel studied by Carleman [3] under very general conditions. Our program here is to consider separately the cases $H(\vert s - t \vert)$ and $H(s + t)$, and to obtain more detailed results by the imposition of additional restrictions. The separation of the two cases is justified by the comparative simplicity of the first, and the dearth of important examples of the second. The restrictions imposed still preserve a class of functions wide enough to enable us to treat adequately the special cases just listed.

The problems to be studied group themselves into three subdivisions. The first of these is an examination of the spectral structure of the transformations generated by the kernels. What is wanted here is an explicit determination of the spectrum, the resolvent and the resolution of the identity. This will be accomplished by the use of standard devices.

Our second problem is suggested by the following considerations: The special transforms

$$f(s) = \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} e^{-(s-t)^2}\varphi(t)\, dt$$

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