CONTACT TRANSFORMATIONS

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The present paper gives a formalism for non-homogeneous contact transformations, defining tensors and quasi-tensors with respect to these transformations. Scalar invariants of contact are obtainable from both kinds of tensors by the process of contraction. As examples we construct several such tensors which are fundamental in the theory of contact transformations, and each of which yields a characterization of these transformations.

1. **Tensors.** Let x^0, x^i, p_i $(i, j = 1, \cdots, n)$ be $2n + 1$ independent variables and x^{*0}, x^{*i}, p_i^* $2n + 1$ functions of them. The passage from the former to the latter is a non-homogeneous contact transformation if

$$dx^{*0} - p_i^* \, dx^{*i} = \rho(dx^0 - p_i \, dx^i) \qquad (\rho \neq 0). \tag{1}$$

We call ρ the *factor* of the contact transformation. Putting

$$p_i = x^{n+i}, \qquad p_i^* = x^{*n+i}, \tag{2}$$

and introducing Greek indices for the range $0, 1, \cdots, 2n$, we may write (1) in the form

$$q_\alpha^* \, dx^{*\alpha} = \rho q_\alpha \, dx^\alpha, \tag{3}$$

where

$$\left\{ \begin{array}{lll} q_0 = -1, & q_i = p_i, & q_{n+i} = 0; \\ q_0^* = -1, & q_i^* = p_i^*, & q_{n+i}^* = 0. \end{array} \right. \tag{4}$$

Since the $2n + 1$ variables x^α are independent by hypothesis, (3) may be written

$$q_\lambda^* \frac{\partial x^{*\lambda}}{\partial x^\alpha} = \rho q_\alpha . \tag{5}$$

If we differentiate (5) with respect to x^β, then interchange α and β, and then subtract, the result is of the form

$$b_{\lambda\mu}^* \frac{\partial x^{*\lambda}}{\partial x^\alpha} \frac{\partial x^{*\mu}}{\partial x^\beta} = \rho b_{\alpha\beta} + q_\alpha \frac{\partial \rho}{\partial x^\beta} - q_\beta \frac{\partial \rho}{\partial x^\alpha}, \tag{6}$$

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