THE CONSTRUCTION OF INTEGRAL QUADRATIC FORMS OF DETERMINANT 1

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1. **Introduction.** The following problem, suggested to us by Paul Erdös, is solved in the first part of this article: to obtain a positive form

$$f = \sum a_{ij}x_ix_j \qquad (a_{ij} = a_{ji} \text{ integers}; i, j = 1, \cdots, n)$$

of determinant 1 and minimum 3. This form will have 24 variables. We shall find also one in 40 variables, of minimum 4. In the latter part of this article we shall develop several interesting facts about integral forms (a_{ii} and $2a_{ij}$ integers) of least possible determinant, and about the minimal constant γ_n.

It is well known that there is only one class of positive n-ary forms f of determinant 1, if $n \leq 7$, namely that containing the form $x_1^2 + \cdots + x_n^2$, see [1; 122–130]. If $n = 8$, Mordell [4] has shown that there are only two classes, one (represented by $x_1^2 + \cdots + x_8^2$) being properly primitive, and the other improperly primitive, and represented by

$$g_8 = \sum_{i=1}^{8} x_i^2 + (\sum_{i=1}^{8} x_i)^2 - 2x_1x_2 - 2x_2x_8 . \tag{1}$$

If $n = 9$, 10, or 11, Chao Ko [2] has shown that there are only two classes, represented by the (properly primitive) forms $x_1^2 + \cdots + x_n^2$ and $g_8 + x_9^2 + \cdots + x_n^2$. Each of these forms for $9 \leq n \leq 11$ has minimum 1. For $n = 12$, 14, and 15, he has given forms of minimum 2. According to Erdös, Chao Ko found also a form in $8n + 4$ variables, of determinant 1 and minimum 2, and such that the least odd number represented is $2n + 1$. However, he was unable to construct a form of determinant 1 and minimum greater than 2.

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