SUFFICIENCY CONDITIONS FOR THE PROBLEM OF LAGRANGE

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Recently, E. J. McShane [5] published a paper on sufficient conditions for weak relative minima. The purpose of the present paper is to extend those results to semi-strong relative minima. We also obtain conditions sufficient for strong relative minima for particular types of integrands.

1. **Statement of the problem.** For simplicity of notation we shall consider the Lagrange problem in non-parametric form with fixed end points. On an open point set R of points $(x, y, r) = (x, y^1, \cdots, y^n, r^1, \cdots, r^n)$ in $(2n+1)$-dimensional space we are given functions

$$f(x, y, r),\ \varphi^\beta(x, y, r) \qquad (\beta = 1, \cdots, m < n)$$

defined and of class C^2. An element (x, y, r) is *admissible* if it belongs to R and satisfies the equations

$$\varphi^\beta(x, y, r) = 0 \qquad (\beta = 1, \cdots, m). \tag{1.1}$$

Let C be an arc with the representation

$$C: y^i = y^i(x) \qquad (a \leq x \leq b; i = 1, \cdots, n). \tag{1.2}$$

We say C is an *admissible arc* provided the functions $y^i(x)$ are absolutely continuous on the interval $[a, b]$, the element $(x, y(x), y'(x))$ is admissible for almost all x in $[a, b]$, and the end conditions

$$y^i(a) = Y_1^i, \qquad y^i(b) = Y_2^i \qquad (i = 1, \cdots, n), \tag{1.3}$$

where Y_1^i and Y_2^i are constants, are satisfied.

The problem of Lagrange in non-parametric form is the problem of minimizing the functional

$$J(C) \equiv \int_a^b f(x, y, y')\, dx \tag{1.4}$$

in the class of all admissible arcs.

We shall denote the length of a vector by enclosing the vector between vertical bars; thus

$$| y_1 - y_2 | = \{\sum_{i=1}^n (y_1^i - y_2^i)^2\}^{\frac{1}{2}}.$$

Let us suppose that C_0 is an arc of class C^1 given by the equations

$$C_0 : y^i = y_0^i(x) \qquad (a \leq x \leq b), \tag{1.5}$$

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