THE ABSOLUTE CESÀRO SUMMABILITY OF TRIGONOMETRICAL SERIES

By Fu Traing Wang

1. The absolute Cesàro summability of series has been defined by Kogbetliantz [9] and treated by many other authors; it is related to Cesàro summability as absolute convergence is related to ordinary convergence. Bosanquet [1], [2], [3] applied this kind of summability to Fourier series and proved that the property of bounded variation of the Cesàro mean of the function is the necessary and sufficient condition that its Fourier series be absolutely Cesàro summable at a point. Hyslop [6], [7], Cooper [5], Chow [4], and Randels [12] discussed the analogous problem for derived Fourier series, conjugate series, Fourier integrals and power series.

Put

$$A_n = A_n(x) = A_n \cos nx + b_n \sin nx,$$

and

$$\sigma_n^\alpha = \frac{1}{\binom{n+\alpha}{n}} \sum_{\nu=0}^{n} \binom{n-\nu+\alpha}{n-\nu} A_\nu .$$

A series $\sum A_n$ is said to be absolutely summable (C, α) or, simply, summable $| C, \alpha |$ provided the series $\sum (\sigma_n^\alpha - \sigma_{n-1}^\alpha)$ is absolutely convergent. The object of this paper is to prove the following theorem and find its best possible condition.

Theorem. *If the series*

$$\sum_{n=2}^{\infty} (a_n^2 + b_n^2)(\log n)^{1+\epsilon} \qquad (\epsilon > 0) \tag{1.1}$$

is convergent, then the trigonometrical series

$$\sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{1.2}$$

is summable $| C, \alpha |$ $(\alpha > \frac{1}{2})$ *almost everywhere.*

First, we can prove that the trigonometrical series

$$\sum_{n=2}^{\infty} (n \log n)^{-1} \cos 2^n x$$

when $\sum_{n=2}^{\infty} a_n^2 \log n$ converges is non-absolutely Cesàro summable at a set of points

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