THE RECIPROCAL OF CERTAIN SERIES

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1. **Introduction.** This paper is concerned with properties of the coefficients in the reciprocal of series of the type

$$f(u) = \sum_{i=0}^{\infty} (-1)^i \frac{A_i}{F_i} u^{p^{ni}} \qquad (A_0 = 1), \tag{1.1}$$

where

$$F_i = [i]F_{i-1}^{p^n}, \qquad [i] = x^{p^{ni}} - x, \qquad F_0 = 1,$$

and the $A_i$ are arbitrary polynomials in the indeterminate $x$ with coefficients in $GF(p^n)$. (While convergence questions are of little interest here we remark that for

$$\deg A_i < cip^i \qquad (c < 1),$$

(1.1) converges for all $u$.) We denote the inverse of $f(u)$ by $\lambda(u)$ so that

$$f(\lambda(u)) = u = \lambda(f(u)); \tag{1.2}$$

then in general we can assert only that $\lambda(u)$ is also of the form (1.1), that is,

$$\lambda(u) = \sum_{i=0}^{\infty} \frac{A_i'}{F_i} u^{p^{ni}}, \tag{1.3}$$

where the $A_i'$ are polynomials in $x$. This follows almost immediately from the recursion formula

$$\sum_{i=0}^{m} (-1)^{m-i} \frac{F_m}{F_i F_{m-i}^{p^{ni}}} A_i A_{m-i}'^{p^{ni}} = 0 \qquad \text{for} \quad m > 0,$$

and the fact that the $F$-quotients are integral (that is, polynomials in $x$). For our purpose we shall require somewhat more, namely that $\lambda(u)$ is of the form

$$\lambda(u) = \sum_{i=0}^{\infty} \frac{D_i}{L_i} u^{p^{ni}}, \tag{1.4}$$

where the $D_i$ are integral and

$$L_i = [i]L_{i-1}, \qquad L_0 = 1;$$

this is equivalent to requiring that the $A_i'$ in (1.3) is a multiple of $F_i/L_i$.

We now put

$$\frac{u}{f(u)} = \sum_{m=0}^{\infty} \frac{\beta_m}{g_m} u^m \qquad (p^n - 1 \mid m), \tag{1.5}$$

where $g_m$ is defined by

$$g_m = F_0^{b_0} F_1^{b_1} \cdots F_s^{b_s}$$

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