# CERTAIN QUANTITIES TRANSCENDENTAL OVER $GF(p^n, x)$

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1. **Introduction.** Let $GF(q)$, $q = p^n$, denote a fixed finite, Galois, field of order $q$; let $x$ be an indeterminate over the field $GF(q)$. If $x$ is adjoined to the field $GF(q)$, a new field $GF(q, x)$ is obtained. We are interested here in the nature of certain quantities over the field $GF(q, x)$. "Transcendental" throughout this paper will mean "transcendental over $GF(q, x)$".

Certain polynomials[1] in $GF(q)$ and certain functions connected with the polynomials in $GF(q)$ are of particular interest. Place

$$[k] = x^{q^k} - x,$$

$$F_k = [k][k-1]^q \cdots [1]^{q^{k-1}},$$

$$L_k = [k][k-1] \cdots [1],$$

$$F_0 = L_0 = 1.$$

If $\psi_k(t) = \prod (t - E)$, extended over all polynomials $E(x)$ of $GF(q)$ of degree $<k$, where $k$ is an arbitrary positive integer, then

$$\psi_k(t) = \sum_{j=0}^{k} (-1)^j \frac{F_k}{F_j L_{k-j}^{q^j}} t^{q^j}.$$

The function

$$\psi(t) = \sum_{k=0}^{\infty} \frac{(-1)^k t^{q^k}}{F_k} \tag{1.1}$$

has the property that

$$\psi(E\xi) = 0 \tag{1.2}$$

for all polynomials $E$ in $GF(q)$ and for a fixed

$$\xi = \lim_{k\to\infty} \frac{[1]^{q^k/(q-1)}}{L_k}. \tag{1.3}$$

Also, for a polynomial $M$ of degree $m$,

$$\psi(Mt) = \sum_{j=0}^{m} \frac{(-1)^j}{F_j} \psi_j(M)\psi^{q^j}(t). \tag{1.4}$$

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[1] For the properties of the polynomials and functions stated below see L. Carlitz, *On certain functions connected with polynomials in a Galois field*, Duke Mathematical Journal, vol. 1(1935), pp. 137–168. Other references are given there.