

MONOTONE TRANSFORMATIONS OF NON-COMPACT TWO-DIMENSIONAL MANIFOLDS

BY VENABLE MARTIN

1. Introduction. The object of this paper is to give an extension of results of Roberts and Steenrod.¹ The generalization in Part I over the results of Roberts and Steenrod consists in removing the assumption of compactness on the manifold, but replacing it by the assumption that the continua are compact. In this case we obtain a complete characterization of the image space of the manifold. Specifically, we define an A -space with identifications (see §§7 and 10 below) and prove that if G is an upper semi-continuous collection of compact continua filling a 2-manifold, then G is an A -space with identifications; and conversely, if S is an A -space with identifications, then there is a 2-manifold M and an upper semi-continuous collection G of compact continua filling M such that G is homeomorphic to S (see Theorem 5, Part I). In addition, we are able to prove under more restrictive hypotheses that G is a manifold (see Theorems 1 and 2), and under still different hypotheses that G is an A -space without identifications (see Theorems 3 and 4).

In Part II neither the manifold nor the continua are assumed to be compact, but the image space is assumed to be metric and the characterization of the image space is effected only in the case in which the manifold has a finite 1-dimensional Betti number. Moreover, we do not show that the characterization is complete in the sense that any space satisfying the restrictions of the characterization is a monotone image of a 2-manifold.

Part I

2. Notation. Throughout this paper M will denote a 2-dimensional manifold without boundary; G will denote an upper semi-continuous collection of continua filling M ; G will also be used to denote the topological space whose points are the continua of this collection, with an element g of G defined as a limit element of a sequence of elements g_1, g_2, \dots of G if and only if there is a point of g which is a limit point of the point set $g_1 + g_2 + \dots$, where $g_i \neq g$.

Received September 2, 1940; presented in part to the American Mathematical Society, December 29, 1938. The author wishes to acknowledge the assistance of Professor J. H. Roberts in the writing of this paper.

¹ J. H. Roberts and N. E. Steenrod, *Monotone transformations of two-dimensional manifolds*, Annals of Mathematics, vol. 39(1938), pp. 851-862. This paper will be cited as MT. As part of the introduction to the present article we assume a reading of the introduction and of the statements of the lemmas and theorems of MT. In particular in Part I we shall speak of upper semi-continuous collections of continua or of monotone transformations as is convenient, without always calling attention to the equivalence of the two points of view.