THE JACOBI CONDITION FOR EXTREMALOIDS

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In a recent paper McShane discussed the Jacobi condition for extremals in a manner which permitted the simultaneous consideration of the parametric and non-parametric problems.[1] It is the purpose of this note to formulate the Jacobi condition for extremaloids in a similar fashion. In order to obtain sufficient flexibility at corners we find it desirable to alter McShane's (Bliss') definition of normality of solutions of the Jacobi equations.[2] No attempt is made to discuss questions of tensor invariance.[3]

A knowledge of the details of McShane's paper, particularly §§2, 4, 5, and 11, is presupposed.

We begin with the consideration of a non-singular extremaloid g: $\gamma^i(t)$ $(i = 1, \cdots, n;\ t_1 \leqq t \leqq t_2)$ with corners at $t = x_\theta$ $(\theta = 1, \cdots, r)$ at which $\Omega_0 \neq 0$, where[4]

$$\Omega_0(t) = f_{y^i}(t^+)\dot{\gamma}^i(t^-) - f_{y^i}(t^-)\dot{\gamma}^i(t^+).$$

In our formulation of the Jacobi condition we shall use *accessory pseudo-extremaloids* which consist of functions $u^i(t)$, $\tau(t)$ $(t_1 \leqq t \leqq t_2)$ with the following properties:

(1) The functions $u^i(t)$, $\tau(t)$ are of class C^2 between corners[5] of g.

(2) The functions $u^i(t)$ satisfy the Jacobi equations between corners of g.

(3) The functions $u^i + \tau\dot{\gamma}^i$, $\Omega_{\rho^i}(u, \dot{u}) + \tau f_{y^i}$, τ are continuous on the whole interval t_1t_2.

We shall see that the values of the function $\tau(t)$ between corners of g are unimportant.

We shall suppose that there is a vector $p(t)$ with the properties:

(1) Each $p_i(t)$ is of class C^2 between corners of g.

(2) $p_i(t)\dot{\gamma}^i(t) \neq 0$ $(t_1 \leqq t \leqq t_2)$.

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[1] E. J. McShane, *The Jacobi condition and the index theorem in the calculus of variations*, this Journal, vol. 5(1939), pp. 184–206.

[2] The referee has shown that we may retain McShane's definition if we are willing to employ still more general accessory pseudo-extremaloids. I am indebted to the referee for additional suggestions which led to a more elegant formulation.

[3] This is not a vital omission. It may be shown, by a simple modification of a proof of M. Morse (*Calculus of Variations in the Large*, American Mathematical Society Colloquium Publications, vol. 18, New York, 1934, p. 109), that there is a neighborhood of a given extremaloid which can be represented by a single coördinate system.

[4] Right and left limits of a function will be indicated by attaching the symbols + and − to the variable.

[5] For brevity's sake we shorten the phrase "of class C^2 between corners of g and having unique right and left limits at corners of g" to "of class C^2 between corners of g".