A REMARK ON THE NORMAL DECOMPOSITIONS OF GROUPS

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In a recent paper[1] I have considered the representations of a group G as the union of normal subgroups

$$G = [A_1, A_2, \cdots, A_n], \tag{1}$$

where the A_i are normally indecomposable in G; i.e., they are not the union of two proper subgroups which are normal in G.

The question arises whether the A_i may be normally decomposable in themselves, i.e., whether there exists a representation

$$A = [B_1, B_2, \cdots, B_l], \tag{2}$$

where the B_i are proper normal subgroups in A. We shall prove the following

Theorem.[2] *A component A_i of non-Abelian type in a normal indecomposable representation* (1) *is also indecomposable in itself.*

To prove this theorem let

$$B_i^{(1)}, B_i^{(2)}, \cdots$$

be the various conjugates of a B_i occurring in some representation (2). All these conjugates are also normal in A, and their union is normal in G. Since A is normally indecomposable in G, this means that there exists one B in (2) such that

$$A = [B^{(1)}, B^{(2)}, \cdots]$$

is the union of indecomposable conjugate groups.[3]

Now let $N^{(i)}$ be the unique maximal normal subgroup of A contained in $B^{(i)}$. All $N^{(i)}$ are conjugate and the simple groups

$$L_i = B^{(i)}/N^{(i)}$$

are all isomorphic. Furthermore, the union

$$C_A = [N^{(1)}, N^{(2)}, \cdots]$$

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[1] O. Ore, *Structures and group theory*, II, this Journal, vol. 4(1938), pp. 247–269.

[2] In the paper mentioned above it was indicated (p. 260, lines 18–21) that such a theorem could be proved. In this statement the condition that A should be of non-Abelian type had, however, inadvertently been omitted.

[3] Of course not all conjugates of the indecomposable B need to appear in the reduced form of this representation, but we assume that the descending chain condition holds for the normal subgroups in A such that the reduced representation contains only a finite number of components.