

CERTAIN EQUIVALENCES AND SUBSETS OF A PLANE

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In the following paragraph is stated a sequence of propositions which are known to hold true in the plane and in certain other spaces.¹ It is the purpose of this paper to study the relations that exist between these propositions in spaces which satisfy R. L. Moore's Axioms 0-4.² For example, it will be shown that in any space satisfying these axioms, each proposition is a consequence of each succeeding proposition and that those propositions of like number are equivalent. Furthermore, in any space which satisfies Moore's Axioms 0-4 and the axiom stated in the second section of this paper, all of these propositions are equivalent. A space exists, however, as is shown by an example, in which all of these propositions are false but which satisfies Axioms 0-4 and the author's axiom of §2 and which is a subset of a plane.

Let α denote the following sequence of propositions:

PROPOSITION 1. *If A and B are two distinct points and neither of the two mutually exclusive closed and compact point sets H and K separates A from B , then $H + K$ does not separate A from B .*

PROPOSITION 2a. *If A and B are distinct points, there exists a simple closed curve separating A from B .*

PROPOSITION 2b. *If the common part of the closed and compact point sets H and K is a continuum and neither H nor K separates the point A from the point B , then $H + K$ does not separate A from B .*

PROPOSITION 2c. *No arc separates space.*

PROPOSITION 3a. *If A and B are distinct points, $A + B$ is a subset of a simple closed curve.*

PROPOSITION 3b. *Space contains no end points.*

PROPOSITION 3c. *If P is a point, there exists a simple closed curve containing P .*

PROPOSITION 4a. *Space contains no local cut points.*

PROPOSITION 4b. *If P is a point of a connected domain, then $D - P$ is connected.*

PROPOSITION 5. *If A is a point of a region R and B is a point distinct from A , there exists in R a continuum T separating A from B .*

PROPOSITION 6a. *If A is a point of a region R and B is a point distinct from A , then R contains a compact continuum separating A from B .*

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¹ See Chapter IV of R. L. Moore's *Foundations of Point Set Theory*. This book will hereinafter be referred to as *Foundations*. For the definition of certain terms and phrases used without definition in this paper the reader is referred to *Foundations*.

² Loc. cit.