THE ANALYTIC PROLONGATION OF A MINIMAL SURFACE OVER A RECTILINEAR SEGMENT OF ITS BOUNDARY

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1. Schwarz proved the theorem that if a minimal surface contains a straight line in its interior, then this straight line must be an axis of symmetry of the surface.[1] All proofs so far given of this theorem make essential use of the supposition of the *interior* position of the line with respect to the minimal surface.

For certain interesting applications (see §6) it is important to have stronger information. Suppose the straight line segment l is part of the *boundary* of a portion M of a minimal surface. Let M be rotated about l through 180°, producing M'. Is then M' the *analytic prolongation* of M across l?

In other words, can symmetry with respect to a straight line be employed as a principle for the analytic prolongation of a minimal surface, and not merely figure as a property of the surface when already prolonged so as to contain the straight line in its interior?

The present paper gives a proof of the validity of the stated prolongation principle. In §6 an application is presented to the construction of a minimal surface bounded by two given interlacing circles. §7 poses the general problem of the analytic prolongation of a minimal surface across an analytic arc.

2. Let H denote the *interior* of the upper half of the (u, v)-plane $(v > 0)$, while ab denotes an *open* segment $(a < u < b)$ of the axis $v = 0$. Suppose that $x(u, v)$, $y(u, v)$, $z(u, v)$ are any three functions which have the following properties:

(i) they are defined and continuous on $H + ab$;

(ii) they are harmonic in H:

$$x_{uu} + x_{vv} = 0, \qquad y_{uu} + y_{vv} = 0, \qquad z_{uu} + z_{vv} = 0; \tag{1}$$

(iii) they obey throughout H the relations

$$x_u^2 + y_u^2 + z_u^2 = x_v^2 + y_v^2 + z_v^2, \tag{2}$$

$$x_u x_v + y_u y_v + z_u z_v = 0; \tag{3}$$

(iv) for (u, v) on ab, i.e., $a < u < b$, $v = 0$, the point of coördinates

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[1] H. A. Schwarz, *Gesammelte Mathematische Abhandlungen*, vol. 1, p. 181. For another proof, see the tract by T. Radó, *On the Problem of Plateau*, 1933, p. 30.