THE SUMMABILITY OF EXPONENTIAL AND FACTORIAL SERIES

By Tomlinson Fort

The summability of ordinary factorial series by Cesàro means has been studied by Bohr.[1] However, nothing seems to have been done since the appearance of Bohr's paper. In §§2 and 3 of the present paper a study is made of what is called R-summability of general factorial series and a type of related exponential series. A general theorem on R-summability is proved in §1. This is one of the most interesting theorems of the paper.

1. R-**summability**. Let there be given a sequence $0 < \lambda_n \to \infty$. Let

$$\sigma_n = \sum_{n=1}^{n} \frac{1}{\lambda_n}$$

and let $\sigma_n \to \infty$. Also let there be given a series

$$\sum_{n=1}^{\infty} a_n(z). \tag{1}$$

Let

$$S_n^{(0)}(z) = \sum_{n=1}^{n} a_n(z)$$

and

$$S_n^{(k)}(z) = \frac{1}{\sigma_n} \sum_{n=1}^{n} \frac{1}{\lambda_n} S_n^{(k-1)}(z), \qquad n > 0.$$

Definition.[2] *We call* $S_n^k(z)$ *the k-th R-mean for series* (1). *If* $\lim_{n\to\infty} S_n^{(k)}(z)$ *exists and equals* $S(z)$, *we say that* (1) *is summable* $R[k, \lambda]$ *to* $S(z)$. *Summation is said to be uniform over a set* P *in case* $S_n^{(k)}(z)$ *approaches its limit uniformly over* P.

We note without proof the following three readily established theorems with reference to R-summability.

Theorem A. *If a series is uniformly summable* $R[k - 1, \lambda]$ *over* P *to* $s(z)$, *then it is uniformly summable* $R[k, \lambda]$ *over* P *to* $s(z)$.

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[1] Gött. Nach., 1909, p. 260.

[2] M. Riesz (Comptes Rendus, vol. 149, p. 18) introduces weighted means as a generalization of summability by the method of the arithmetic mean of the first order. He generalizes to his "typical means" of arbitrary order. R-summability as defined here generalizes Riesz means of the first order by simple iteration as the Hölder method generalizes the ordinary arithmetic mean of the first order.