INFINITE SYSTEMS OF LINEAR EQUATIONS AND EXPANSIONS OF ANALYTIC FUNCTIONS

By P. W. Ketchum

Introduction. In the present article we emphasize a mutual relationship existing between the theory of the solution of infinite systems of linear equations in an infinity of unknowns and the theory of the expansion of an analytic function in a series of analytic functions. In the first two sections we show how known expansion theorems can be used to give theorems on the solution of infinite systems of equations. In particular, an expansion theorem of Birkhoff[1] yields a theory similar to that of von Koch[2] on normal determinants.

In the remainder of the paper we apply known theorems, or suitable modifications of known theorems, on infinite systems of linear equations to new situations so as to obtain generalizations of certain types of expansion theorems. These expansions are similar in character to those of Pincherle,[3] who was the first to show that any function $f(x)$ analytic at $x = 0$ can be expanded in a series of the form

$$f(x) = \sum_{m=0}^{\infty} c_m x^m G_m(x), \qquad G_m(0) = 1, \tag{1}$$

provided the functions $G_m(x)$ are analytic and uniformly bounded in some neighborhood of $x = 0$.

We have adopted in this part of the paper a point of view initiated by I. M. Sheffer,[4] according to which a known expansion theorem is generalized by replacing the functions, in terms of which we are expanding, by linear combinations of those same functions, the coefficients of these linear combinations being restricted by appropriate conditions so that the resulting sums will be "close to" the original functions.

In the expansions of Pincherle, the n-th term has a zero of order n at the origin. Our expansions differ from those of Pincherle principally in that the n-th term has n zeros not necessarily coincident.

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[1] Comptes Rendus, vol. 164(1917), pp. 942–945.

[2] See F. Riesz, *Les Systèmes d'Équations Linéaires à une Infinité d'Inconnues*, Paris, 1913, Chapter II.

[3] Memorie della Accademia delle Scienze dell' Istituto di Bologna, (4), vol. 3(1881), pp. 151 ff.

[4] American Journal of Mathematics, vol. 57(1935), pp. 587–614.