# $(n-1)$-DIMENSIONAL CHARACTERISTIC STRIPS OF A FIRST ORDER EQUATION AND CAUCHY'S PROBLEM

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Consider the first order partial differential equation

$$F(x^\alpha \mid z \mid p_\alpha) = 0 \qquad (p_\alpha = \partial z/\partial x^\alpha) \tag{a}$$

in one unknown $z$ and $n$ independent variables $x^\alpha$ ($n \geqq 3$). The purpose of the present paper is to generalize to the case of more than two independent variables the usual geometrical discussion of Cauchy's problem showing the manifolds for which the problem is indeterminate.[1] In doing this we introduce the concept of an $(n-1)$-dimensional characteristic strip and study its relation to the one-dimensional characteristic strips.

1. We first recall the geometrical approach to the one-dimensional characteristic strip. If we assume that the space $S^{n+1}$ with coördinates $x^1, \cdots, x^n$, $z$ is Euclidean with rectangular Cartesian coördinates, the problem of integrating the equation (a) is that of determining a hypersurface[2]

$$z = z(x^1, \cdots, x^n) \tag{1.1}$$

in $S^{n+1}$ such that the direction ratios $p_1 : \cdots : p_n : -1$ of the normal to (1.1) satisfy the condition (a) at each point $P$ of (1.1). The geometrical configuration consisting of a point $P(x^1, \cdots, x^n, z)$ and a hyperplane passing through $P$, namely,

$$Z - z = p_\alpha(X^\alpha - x^\alpha), \tag{1.2}$$

in called an element. In general, the integral elements at $P$, i.e., those which are possible tangent hyperplanes to integral hypersurfaces at $P$, envelope a hypercone $T$ with vertex at $P$. It then follows easily that an integral element (1.2) is tangent to the hypercone $T$ along the generator given by[3]

$$\frac{X^1 - x^1}{F_{p_1}} = \cdots = \frac{X^n - x^n}{F_{p_n}} = \frac{Z - z}{p_\alpha F_{p_\alpha}}. \tag{1.3}$$

Received June 16, 1937; developed in part while the author was a National Research Fellow and mentioned in a paper presented to the American Mathematical Society, February 23, 1935. See Abstract 41-3-103, Bull. Amer. Math. Soc., vol. 41 (1935), p. 182.

[1] See, for example, E. Goursat, *Cours d'Analyse Mathématique*, Tome II, 1924, Chapter 22. We do not discuss the regularity requirements on $F$ or the initial manifold. For a treatment of this question in the case of two independent variables the reader is referred to G. A. Bliss, Princeton Colloquium Lectures, Amer. Math. Soc., 1913, p. 98.

[2] In $S^{n+1}$ we shall call an $n$-dimensional spread, a hypersurface, and an $(n-1)$-dimensional spread, an edge. For linear spreads we shall use the terminology hyperplane and plane edge.

[3] Cf. Goursat, loc. cit., p. 616.