ANALYTIC FUNCTIONS OF ABSOLUTELY CONVERGENT GENERALIZED TRIGONOMETRIC SUMS

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1. **Introduction.** It has been shown by Wiener[1] that a nowhere vanishing periodic function with an absolutely convergent Fourier series has a reciprocal whose Fourier series also converges absolutely. Lévy[2] has pointed out that this result can be extended from reciprocals to general analytic functions. Thus if $f(x)$ is periodic and never zero and has an absolutely convergent Fourier series, it follows that $F[f(x)]$ also has an absolutely convergent Fourier series provided that $F(z)$ is analytic and single valued whenever $z = f(x)$. One of the results of this paper (Theorem I) shows that these results are true in n or even $\aleph_0$ dimensions. This is accomplished by carrying through Wiener's proof with the necessary modifications to take care of dimensionality.

One might reasonably ask whether this result can be extended from periodic to almost periodic functions. A partial answer to this question has been given by Bochner,[3] who has shown that reciprocals of trigonometric polynomials which are bounded away from zero on the real axis have absolutely convergent Fourier series. It is shown in the present paper[4] that the theorem is true not only for trigonometric polynomials, but also for absolutely convergent infinite trigonometric sums. No further hypothesis is required; so the exponents are altogether unrestricted and may be any countable set of real numbers. Moreover this result is true not only for reciprocals, but for all analytic functions; and it holds in n or even $\aleph_0$ dimensions. Thus the final result of the paper is

Theorem II. *Let $f(x_1, x_2, \cdots)$ be an almost periodic function with an absolutely convergent Fourier series, and let R be the closure of its set of values. Then if $F(z)$ is a function analytic over an open set S containing R, it follows that $F[f(x_1, x_2, \cdots)]$ is an almost periodic function with an absolutely convergent Fourier series.*

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[1] N. Wiener, *Tauberian theorems*, Ann. of Math., (2), vol. 33 (1932), pp. 1–100; p. 14.

[2] P. Lévy, *Sur la convergence absolue des séries de Fourier*, C. R. Acad. Sci., Paris, vol. 196 (1933), pp. 463–464.

[3] S. Bochner, *Beitrag zur absoluten Konvergenz fastperiodischer Fourierreihen*, Jahresbericht der Deutschen Math. Ver., vol. 39 (1930), pp. 52–54.

[4] After this paper had been submitted for publication, the author learned that his main theorem (without the extension to analytic functions or to more than one dimension) has been proved independently by H. R. Pitt. Apparently Pitt's work was done somewhat earlier than the author's, though it was not submitted for publication until about the time the present paper was accepted for publication. It will appear in an early issue of the Journal of Mathematics and Physics, Massachusetts Institute of Technology.