UNIFORMITY PROPERTIES IN TOPOLOGICAL SPACE SATISFYING THE FIRST DENUMERABILITY POSTULATE

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Theorems involving convergence, completeness and uniform continuity are consequences of what may be called the uniformity properties which inhere in metric spaces. It is perhaps of some interest to seek in spaces which are not metrizable similarly effective uniformity properties, in terms of which the theorems referred to may be reformulated.[1] It is the purpose of this note to do this. The spaces considered are the topological spaces of Hausdorff satisfying the first denumerability postulate, namely, that to each $p \subset S$ the topology at p is determined by a sequence of neighborhoods $U_n(p)$. The uniformizing entity is the class $[U_n]$ of all $U_n(p)$ for fixed n and all $p \subset S$. We make the following definitions:

1. A sequence $p_k \subset S$ is a *Cauchy sequence* if for each n there is a k_n and a $q_n \subset S$ such that $p_k \subset U_n(q_n)$ if $k > k_n$.
2. A space S is *complete* if every Cauchy sequence has a limit.
3. A set $M \subset S$ is *totally bounded* if for each n there are points

$$p_{n,1},\ p_{n,2},\ \cdots,\ p_{n,m_n}$$

such that

$$M \subset \sum_{i=1}^{m_n} U_n(p_{n,i}).$$

4. A function f on $M \subset S$ to S' is *uniformly continuous* on M if for each n there is an $m(n)$ such that if $p \subset M$

$$f[MU_{m(n)}(p)] \subset U'_n(f(p)).$$

It is clear that these definitions become the usual ones for metric space with spherical neighborhoods.

The justification for these generalizations is to be sought in the theorems that a set M in a complete space is compact if and only if it is totally bounded, and that if F is uniformly continuous on $M \subset S$ to a complete space S', then there is a function F^* on M to S' identical with F on M and continuous on $\bar{M}$.

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[1] The relation between completeness and compactness is discussed by J. von Neumann, Annals of Math., vol. 36 (1935). Recent notes on the subject have been published by Garrett Birkhoff, Annals of Math., vol. 38 (1937), pp. 57–60, and by L. M. Graves, Annals of Math., vol. 38 (1937), pp. 61–64.