CONCERNING APPELL SETS AND ASSOCIATED LINEAR FUNCTIONAL EQUATIONS

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Introduction. We have elsewhere considered the linear difference equation with constant coefficients[1]

$$\sum_{j=1}^{k} a_j y(x+\omega_j) = F(x), \tag{1}$$

from the point of view of a local solution. That is, assuming only that $F(x)$ is analytic about a point, we have shown that $y(x)$ exists satisfying (1) in the neighborhood of this point. Now (1) is a particular case of the general linear differential equation of infinite order[2]

$$L[y(x)] \equiv \sum_{n=0}^{\infty} a_n y_{(x)}^{(n)} = F(x), \tag{2}$$

where if we set

$$L(t) \sim \sum_{0}^{\infty} a_n t^n \tag{3}$$

and call $L(t)$ the *generating function* for the operator $L[y]$, then the generating function for (1) is

$$L(t) = \sum_{j=1}^{k} a_j e^{\omega_j t}.$$

This suggests the possibility of developing a local theory for equation (2), at least when $L(t)$ is suitably restricted.

The solubility of equation (2) is linked with the problem of expanding $F(x)$ in a series of Appell polynomials $\{P_n(x)\}$ generated by $L(t)$; i.e., where the sequence $\{P_n(x)\}$ is defined by

$$L(t)e^{tx} \sim \sum_{n=0}^{\infty} P_n(x)t^n. \tag{4}$$

We see this formally from the fact that

$$L\left[\frac{x^n}{n!}\right] = P_n(x), \tag{5}$$

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[1] Sheffer, Transactions of the American Mathematical Society, vol. 39 (1936), pp. 345–379, and vol. 41 (1937), pp. 153–159.

[2] For an investigation of equation (2) from another point of view see H. T. Davis, American Journal of Mathematics, vol. 52 (1930), pp. 97–108.