THE CHARACTERISTIC ROOTS OF A MATRIX

By W. V. Parker

1. **Introduction.** If A is a square matrix of order n and I is the unit matrix, the equation obtained by equating to zero the determinant $| A - \lambda I |$ is called the *characteristic equation* of A. The roots of this equation are called the *characteristic roots* of A.

If A is a matrix of a particular type, certain definite statements may be made concerning the nature of its characteristic roots. For example, if A is Hermitian its characteristic roots are all real. While it is not possible to make any definite statement about the nature of the characteristic roots for the general matrix, several authors have given upper limits to the roots. The first of these limits seems to have been given by Bendixson[1] in 1900. He obtained upper limits for the real and imaginary parts of the characteristic roots of a *real* matrix. In a letter to Bendixson in 1902, Hirsch[2] extended these results to include the case when the elements of A may be complex numbers. Hirsch obtained an upper limit for the characteristic roots as well as for their real and imaginary parts. A limit was also given by Bromwich[3] in 1904. In 1930, Browne[4] obtained limits which do not exceed those previously found and are in general less.

In the present note it is shown that the limit for the characteristic roots can generally be determined to be less than the one given by Browne. A lower limit for the characteristic root of greatest absolute value and an upper limit for the characteristic root of least absolute value for an Hermitian matrix are also found.

Let A' and $\bar{A}$ denote the transpose and conjugate, respectively, of the square matrix A and write

$$B = \frac{A + \bar{A}'}{2}, \qquad C = \frac{A - \bar{A}'}{2i}.$$

It is evident that B and C are Hermitian; that is, $B = \bar{B}'$ and $C = \bar{C}'$. A theorem given by Browne may be stated as follows:

Browne's Theorem. *If R_i, R_i', and R_i'' are the sums of the absolute values of the elements in the i-th row of the matrices A, B, and C, respectively, and if T_i*

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[1] Bendixson, *Sur les racines d'une équation fondamentale*, Acta Mathematica, vol. 25 (1902), pp. 359–365.

[2] Hirsch, Acta Mathematica, vol. 25 (1902), pp. 367–370.

[3] Bromwich, *On the roots of the characteristic equation of a linear substitution*, Acta Mathematica, vol. 30 (1906), pp. 295–304.

[4] Browne, *The characteristic roots of a matrix*, Bulletin of the American Mathematical Society, vol. 36 (1930), pp. 705–710.