CONNECTIONS BETWEEN DIFFERENTIAL GEOMETRY AND TOPOLOGY

II. CLOSED SURFACES

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1. Introduction. This paper deals with closed 2-dimensional Riemannian manifolds, for brevity designated *closed surfaces*. The properties of a fundamental locus which we call the minimum point locus with respect to a point A, studied in a previous paper[1] by the author for the case of simply-connected analytic surfaces, are determined here for the general class of closed surfaces. The locus in question is defined as the locus m of points M on geodesic rays issuing from a point A, which are the last points along these rays such that the arc AM furnishes an absolute minimum (proper or improper) to the length of arcs joining A to M. In the case of a closed analytic surface S the principal result is that m is a linear graph (i.e., a finite connected 1-dimensional complex) whose one-dimensional Betti number equals the one-dimensional Betti number mod 2 of the surface. A study is made of the parametrization of m by means of θ, the angular coördinate of the geodesic rays through A. It is found that this depends on the orientability of S, and also that the number of values of θ yielding one point of m equals the order[2] of that point in m.

A brief study is also made of non-analytic surfaces. Here we assume, for example, that S is a closed regular manifold of[3] class 5 with a Riemannian line element of class C^4. The locus m turns out to be a continuous curve (not necessarily a linear graph) with the same relation as in the analytic case between the one-dimensional Betti number of m and the one-dimensional Betti number of S, and similar relations among the orientability of S, the parametrization of m by means of θ, and the order of points of m.

In both analytic and non-analytic cases, if we subtract the locus m from the surface S, the result is a single 2-cell with m as its singular boundary, simply covered (except at A) by the geodesic rays through A. Thus is solved the prob-

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[1] See Myers, *Connections between differential geometry and topology, I. Simply connected surfaces*, this Journal, vol. 1 (1935), pp. 376–391. This paper will be referred to as (I). An abstract containing the results of that paper, as well as the results of the present paper in the analytic case, appears in the Proc. Nat. Acad. Sci., April, 1935, under the same title.

[2] By the order of a point P of a continuous curve C we mean the number of 1-cells contained in C which issue from P and are such that no two of them have any point in common but P.

[3] See Veblen and Whitehead, *The Foundations of Differential Geometry*, p. 81.