SOME IRREDUCIBLE MONOMIAL REPRESENTATIONS OF HYPERORTHOGONAL GROUPS

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1. A large number of simple groups of finite order can most easily be defined by means of matrices with coefficients from a finite field, whose characteristic p is a factor of the order of the group. A certain infinite family of these simple groups may be represented by unitary matrices of degree m with coefficients from a finite field $GF(q^2)$ of q^2 elements. Here q is the power p^s of a prime p, and to each number x a conjugate is defined by the relation $\bar{x} = x^q$. Since each x from the $GF(q^2)$ satisfies the equation $x^{q^2} = x$, it follows that $\bar{\bar{x}} = \bar{x}^q = x^{q^2} = x$. By an m-dimensional GF-vector a, we shall mean an ordered set $(a_1, a_2, \cdots, a_m)$ of m numbers from the field $GF(q^2)$, and we shall use the notation $\sum_{i=1}^{m} \bar{a}_i b_i = (a \mid b) = (\overline{b \mid a})$, calling the vectors a and b orthogonal if $(a \mid b) = 0$.

In a recent paper[1] the author has studied some of these simple groups, resolved them into sets of conjugate operations, and found for each group a representation as a permutation group of degree $q^3 + 1$, which was easily reduced into its two irreducible components. In this paper we shall find a set of monomial representations of these groups, also of degree $q^3 + 1$, with complex coefficients, some of which are irreducible, and the rest of which split into two irreducible components. Together these determine more than half of the distinct irreducible representations. With the aid of the familiar relations between characters, we are then able to determine the degrees and most of the characters of all the irreducible representations of these groups.

Let G_m^* be the group of unitary matrices of degree m in this Galois field $GF(q^2)$; that is, the group of those matrices T which leave invariant the form $(x \mid x)$. The matrices have the elements (t_{ij}), where $\sum_{k=1}^{m} \bar{t}_{ik}\, t_{jk} = \sum_{k=1}^{m} \bar{t}_{ki}\, t_{kj} = \delta_{ij}$. In short, T is the transposed matrix of $\overline{T}$. Since its determinant satisfies the equation $T\overline{T} = 1$, it can have one of only $q + 1$ possible values. If we write $a' = aT$ when $a'_j = \sum_{i=1}^{m} a_i t_{ij}$, $(j = 1, 2, \cdots, m)$, we find $(aT \mid bT) = (a \mid b)$ for every matrix T of G_m^*, and for all GF-vectors a and b. Dickson[2] has shown that the order of G_m^* is

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[1] J. S. Frame, *Unitäre Matrizen in Galoisfeldern*, Commentarii Mathematici Helvetici, vol. 7 (1935), p. 94. This paper will be cited here as U.

[2] L. E. Dickson, *Linear Groups*, 1907.