Vol. 104, No. 1 DUKE MATHEMATICAL JOURNAL © 2000

ABEL-JACOBI MAPPINGS AND FINITENESS OF MOTIVIC COHOMOLOGY GROUPS

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CONTENTS

Introduction. Let X be a projective smooth variety over a global field k. Let $\overline{k}$ denote a separable closure of k, and let $\overline{X}$ denote the scalar extension of X to $\overline{k}$. Let $\mathrm{H}^i_{\mathcal{M}}(X, \mathbb{Q}(n))$ denote the motivic cohomology groups of X with $\mathbb{Q}$-coefficients in the sense of Bloch [B3] or Voevodsky [Vo]. For example, we have

$$\mathrm{H}^{2n}_{\mathcal{M}}\big(X, \mathbb{Q}(n)\big) \simeq \mathrm{CH}^n(X)_{\mathbb{Q}},$$

where the right-hand side is the Chow group of codimension n cycles on X tensored with $\mathbb{Q}$. By combining cycle classes from motivic cohomology to continuous étale cohomology of X over k with the Hochschild-Serre spectral sequence

$$\mathrm{H}^r_{\mathrm{cont}}\big(G_k, \mathrm{H}^s_{\text{ét}}\big(\overline{X}, \mathbb{Q}_l(n)\big)\big) \Longrightarrow \mathrm{H}^{r+s}_{\mathrm{cont}}\big(X, \mathbb{Q}_l(n)\big),$$

we get a homomorphism

$$\mathrm{aj}^{i,n}_l : \mathrm{H}^i_{\mathcal{M}}\big(X, \mathbb{Q}(n)\big)_{\mathrm{hom}} \otimes_{\mathbb{Q}} \mathbb{Q}_l \longrightarrow \mathrm{H}^1_{\mathrm{Gal}}\big(G_k, \mathrm{H}^{i-1}_{\text{ét}}\big(\overline{X}, \mathbb{Q}_l(n)\big)\big), \qquad (*)$$

which is called the *l-adic Abel-Jacobi mapping* (cf. §5, (5.1)). The meaning of the subscript hom in the group on the left-hand side is described in the main body of the paper below (cf. appendix, (A.3)). In the case of Chow groups, it means cycles homologically equivalent to zero. Otherwise, it means the full group (cf. §1, Lemma 1.1). The general philosophy of mixed motives of Bloch-Beilinson and the Beilinson-Deligne conjecture on the triviality of motivic 2-extensions imply that these mappings should

Received 7 June 1999. Revision received 1 December 1999.
2000 *Mathematics Subject Classification*. Primary 19F27; Secondary 11G25, 11G35.
Author's work supported by the Japan Society for the Promotion of Science Research Fellowships for Young Scientists.