

# HOLOMORPHIC CURVES AND HAMILTONIAN SYSTEMS IN AN OPEN SET WITH RESTRICTED CONTACT-TYPE BOUNDARY

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**1. Introduction and main results.** Throughout this paper, we fix an integer  $n \geq 2$  and consider the standard symplectic space  $(\mathbb{R}^{2n}, \omega = d\lambda_0)$ , with  $n \geq 2$  and

$$\lambda_0 = \frac{1}{2} \sum_{k=1}^n x_k \wedge dy_k - y_k \wedge dx_k. \quad (1.1)$$

We identify  $\mathbb{R}^{2n}$  with  $\mathbb{C}^n$ , setting  $z_k = x_k + iy_k$ . To each (time-dependent) Hamiltonian  $H \in \mathcal{H}_t = C^\infty(S^1 \times \mathbb{C}^n)$ , we can associate a Hamiltonian vector field given by

$$i_{X_H} \omega = -dH(t, \cdot), \quad (1.2)$$

where  $i_{X_H} \omega$  denotes the contraction of  $\omega$  by  $X_H$ . The flow  $\phi_t^H$  of  $X_H$  is called the Hamiltonian flow of  $H$  and is a symplectic isotopy. We also denote by  $\mathcal{D}$  the group

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