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QUANTUM DETERMINANTAL IDEALS

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Introduction. Fix a base field k. The quantized coordinate ring of $n \times n$ matrices over k, denoted by $\mathcal{O}_q(M_n(k))$, is a deformation of the classical coordinate ring of $n \times n$ matrices, $\mathcal{O}(M_n(k))$. As such, it is a k-algebra generated by n^2 indeterminates X_{ij}, for $1 \leq i, j \leq n$, subject to relations which we state in (1.1). Here, q is a nonzero element of the field k. When $q = 1$, we recover $\mathcal{O}(M_n(k))$, which is the commutative polynomial algebra $k[X_{ij}]$. The algebra $\mathcal{O}_q(M_n(k))$ has a distinguished element D_q, the *quantum determinant*, which is a central element. Two important algebras $\mathcal{O}_q(\mathrm{GL}_n(k))$ and $\mathcal{O}_q(\mathrm{SL}_n(k))$ are formed by inverting D_q and setting $D_q = 1$, respectively.

The structures of the primitive and prime ideal spectra of the algebras $\mathcal{O}_q(\mathrm{GL}_n(k))$ and $\mathcal{O}_q(\mathrm{SL}_n(k))$ have been investigated recently (see, for example, [2], [7], and [10]). Results obtained in these investigations can be pulled back to partial results about the primitive and prime ideal spectra of $\mathcal{O}_q(M_n(k))$. However, these techniques give no information about the closed subset of the spectrum determined by D_q. In this paper, we begin the study of this portion of the spectrum.

In the classical commutative setting, much attention has been paid to *determinantal ideals*: that is, the ideals generated by the minors of a given size. In particular, these are special prime ideals of $\mathcal{O}(M_n(k))$ containing the determinant. Moreover, there are interesting geometrical and invariant theoretical reasons for the importance of these ideals (see, for example, [4]). In order to put our results into context, it may be useful to review some highlights of the commutative theory.

Let $M_{l,m}(k)$ denote the algebraic variety of $l \times m$ matrices over k. For $t \leq n$, the general linear group $\mathrm{GL}_t(k)$ acts on $M_{n,t}(k) \times M_{t,n}(k)$ via

$$g \cdot (A, B) := \left(Ag^{-1}, gB\right).$$

Matrix multiplication yields a map

$$\mu : M_{n,t}(k) \times M_{t,n}(k) \longrightarrow M_n(k),$$

the image of which is the set of matrices with rank at most t.

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