PERTURBATION OF SCATTERING POLES FOR HYPERBOLIC SURFACES AND CENTRAL VALUES OF L-SERIES

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1. Introduction. Let $\Gamma\backslash\mathbb{H}$ be a noncompact hyperbolic surface of finite area. In the analysis of the Laplace-Beltrami operator on it, apart from the L^2 spectrum, a crucial role is played by the scattering poles. They appear in the analog of Weyl's law, in the Selberg zeta function, and in the determinant of the Laplace operator; see [18]. More important is the fact that imbedded eigenvalues become scattering poles under perturbation (see [25]). In this work we study variation formulas arising from perturbations of scattering poles.

A scattering pole is a pole of the analytic continuation of the determinant of the scattering matrix $\Phi(s)$ to the left half-plane $\Re s < 1/2$. These poles show among the poles of the analytic continuation of the resolvent on the same half-plane. But, generally, they are not exactly the same. There can be only finitely many exceptions for $0 \le s < 1/2$, where the resolvent has a pole at s_0, corresponding to a cuspidal eigenvalue $s_0(1-s_0) \in [0, 1/4)$, and where $\det\Phi(s)$ has a zero at s_0. We assume that this does not happen, or, alternatively, we look at scattering poles that do not lie on the interval $[0, 1/2)$.

Let $\mathbf{E}(z,s) = (E_1(z,s), \dots, E_n(z,s))^T$ be the vector of Eisenstein series indexed by the cusps. Let m be the multiplicity of the pole of $\det\Phi(s)$ at s_0. We set $s(\epsilon)$ to be the weighted mean of scattering poles; that is, if $s_1(0) = s_2(0) = \cdots = s_m(0) = s_0$ and if the scattering poles split as $s_1(\epsilon)$, $s_2(\epsilon), \dots, s_m(\epsilon)$, when the perturbation is switched on, then

$$s(\epsilon) = \frac{1}{m}\sum_{i=1}^{m} s_i(\epsilon).$$

The first theorem concerns the first variation of $s(\epsilon)$ at $\epsilon = 0$:

$$\dot{s} = \frac{d}{d\epsilon}s(0).$$

Throughout this work $d\mu$ denotes the invariant hyperbolic measure $dx\,dy/y^2$.

THEOREM 1.1. *Assume all the Eisenstein series have a pole of order at most* 1 *at* s_0. *For a compactly supported perturbation of the metric, the first variation of the*

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