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RIGID LOCAL SYSTEMS, HILBERT MODULAR FORMS, AND FERMAT'S LAST THEOREM

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Contents

Introduction. Historically, two approaches have been followed to study the classical Fermat equation $x^r + y^r = z^r$. The first, based on cyclotomic fields, leads to questions about abelian extensions and class numbers of $K = \mathbb{Q}(\zeta_r)$ and values of the Dedekind zeta-function $\zeta_K(s)$ at $s = 0$. Many open questions remain, such as Vandiver's conjecture that r does not divide the class number of $\mathbb{Q}(\zeta_r)^+$. The second approach is based on modular forms and the study of 2-dimensional representations of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$. Even though 2-dimensional representations are more subtle than abelian ones, it is by this route that Fermat's last theorem was finally proved (cf. [Fre], [Se2], [Ri2], [W3], and [TW]; or [DDT] for a general overview).

This article examines the equation

$$\boxed{x^p + y^q = z^r.} \tag{1}$$

Certain 2-dimensional representations of $\mathrm{Gal}(\bar{K}/K)$, where K is the real subfield of a cyclotomic field, emerge naturally in the study of equation (1), giving rise to a blend of the cyclotomic and modular approaches. The special values $\zeta_K(-1)$, which

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