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EXAMPLES WITH BOUNDED DIAMETER GROWTH AND INFINITE TOPOLOGICAL TYPE

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0. Introduction. In this paper, we construct an example of an open manifold with positive Ricci curvature, infinite topological type, and bounded diameter growth. Let M^n be an open manifold with base point $p \in M^n$. For $r > 0$, denote by $C(p,r)$ the union of the unbounded components of $M^n \setminus \overline{B_r(p)}$. U. Abresch and D. Gromoll [AG] defined diameter growth as

$$\operatorname{diam}(p,r) = \sup \operatorname{diam}\left(\Sigma_k, C\left(p, \frac{3}{4}r\right)\right), \tag{0.1}$$

where the supremum is taken over all components Σ_k of $\partial C(p,r)$ and $\operatorname{diam}(\Sigma_k, C(p, (3/4)r))$ is the diameter of Σ_k calculated with respect to the intrinsic distance of $C(p,(3/4)r)$. Here, we show the following theorem.

THEOREM 0.2. *There exists a complete 4-dimensional manifold with*

(0.3) *positive Ricci curvature,*

(0.4) *infinite topological type, and*

(0.5) *bounded diameter growth.*

This result can be generalized to any dimension greater than 5 by taking products with any compact manifold of positive Ricci curvature. If we take a product with a circle, we get a 5-dimensional example satisfying (0.4) and (0.5) but whose Ricci curvature is nonnegative. Theorem 0.2 is the first example of a manifold satisfying (0.3), (0.4), and (0.5). Previously, J. Sha and D. G. Yang [ShYg] constructed manifolds satisfying (0.3) and (0.4); see also the examples of M. T. Anderson [An]. Examples with Ricci-flat Kähler metrics were later given by Anderson, Kronheimer, and LeBrun [AnKL]. All these examples are collapsing, namely, $\lim_{r\to 0} r^{-n}\operatorname{Vol}(B_r(p)) = 0$. Among them, the smallest diameter growth is due to [ShYg] and is equivalent to $r^{2/3}$. More recently, G. Perelman [P] constructed 4-dimensional compact manifolds with positive Ricci curvature, large volume, and large Betti numbers. We used these constructions in [M2] to give examples of manifolds with (0.3), (0.4), and Euclidean volume growth.

In contrast to the examples of Sha and Yang, the sectional curvature of our example in Theorem 0.2 is not bounded from below. Indeed, according to a result of Abresch

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