Vol. 101, No. 1 DUKE MATHEMATICAL JOURNAL © 2000

ALMOST COMPLEX STRUCTURES ON $S^2 \times S^2$

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1. Introduction. It is well known that every symplectic form on $X = S^2 \times S^2$ is, after multiplication by a suitable constant, symplectomorphic to a product form $\omega^\lambda = (1+\lambda)\sigma_1 + \sigma_2$ for some $\lambda \geq 0$, where the 2-form σ_i has total area 1 on the ith factor. We are interested in the structure of the space $\mathcal{J}^\lambda$ of all C^∞ ω^λ-compatible, almost complex structures on X. Observe that $\mathcal{J}^\lambda$ itself is always contractible. However, it has a natural stratification that changes as λ passes each integer. The reason for this is that as λ grows, the set of homology classes that can be represented by an ω^λ-symplectically embedded 2-sphere changes. Since each such 2-sphere can be parametrized to be J-holomorphic for some $J \in \mathcal{J}^\lambda$, there is a corresponding change in the structure of $\mathcal{J}^\lambda$.

To explain this in more detail, let $A \in H_2(X, \mathbf{Z})$ be the homology class $[S^2 \times \mathrm{pt}]$ and let $F = [\mathrm{pt} \times S^2]$. (The reason for this notation is that we are thinking of X as a fibered space over the first S^2-factor, so that the smaller sphere F is the fiber.) When $\ell - 1 < \lambda \leq \ell$,

$$\omega^\lambda(A - kF) > 0 \quad \text{for } 0 \leq k \leq \ell.$$

Moreover, it is not hard to see that for each such k, there is a map $\rho_k : S^2 \to S^2$ of

Received 13 October 1998.
1991 *Mathematics Subject Classification*. Primary 53C15.
Author partially supported by National Science Foundation grant number DMS 9704825.