ERRATUM FOR "TAUTOLOGICAL CLASSES ON MODULI SPACES OF HYPER-KÄHLER MANIFOLDS"

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Thorsten Beckman and Mirko Mauri have pointed out to us a gap in the proof of [1, Theorem 8.2.1]. We do not know how to correct the entire proof, and we can only recover a weaker statement (see Proposition 1 below). This gap affects the proof of the first of the two main results of [1], namely, Theorem 8.3.1, as well as its weaker versions Theorem 4.3.1 and Theorem 1 of the introduction. More precisely, [1, Theorem 4.3.1] claims that the cohomological tautological conjecture **(CTC)** holds for moduli spaces $\mathcal{F}$ of polarized hyper-Kähler varieties of dimension $2n$ with level structures when

$$\dim \mathcal{F} > 8n.$$

As a corollary, **CTC** was stated to hold for both K3 surfaces (see [1, Theorem 1]), where $n = 1$ and $\dim \mathcal{F} = 19$, and $\mathrm{K3}^{[2]}$-type hyper-Kähler manifolds, where $n = 2$ and $\dim \mathcal{F} = 20$. We explain below that these two corollaries can be saved.

The cohomological version of the generalized Franchetta conjecture Theorem 8.1.1, as well as the weaker version Theorem 1.2.1 stated in the introduction, still hold. A direct consequence of the cohomological generalized Franchetta conjecture is the following proposition, where we keep notation as in [1, Theorem 8.2.1].

PROPOSITION 1

Let α_1 and α_2 be two cycles in $\mathrm{CH}^\bullet(\mathcal{U})$. Assume that the sum of the degrees of α_1 and α_2 is $< \frac{1}{4}\dim \mathcal{F}$. Then the two cup products of $[\alpha_1]$ and $[\alpha_2]$ in $H^\bullet(\mathcal{U}, \mathbb{C})$ associated to the two sides of [1, (8.1)] differ by a class supported on the Noether–Lefschetz locus of $\mathcal{F}$.

It is still unkown if the same statement holds for higher-degree cycles. The problem is that the FKM ring is a subring of the right-hand side of [1, (8.1)], while in the last sentence of the proof of [1, Theorem 8.2.1] we use that it is stable under the ring structure induced by the *left*-hand side of [1, (8.1)].

It therefore remains open whether the two cup products coincide (up to some classes supported on the Noether–Lefschetz locus) on the entire subalgebra of

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