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A condition for the extension of a complex line bundle for a family of Kähler surfaces

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It is well known that the surfaces of degree 3 in a projective 3-space contain straight lines, while some of the surfaces of degree 4 do, and some do not, contain straight lines. In view of this fact, we are led to the following question: Let a differentiable family $\mathcal{V} \to M$ of compact complex analytic manifolds and an analytic submanifold W of V_0 be given. (V_t denotes the member of $\mathcal{V}$ corresponding to $t \in M$.) Then under what condition does there exist a submanifold $\mathcal{W}$ of $\mathcal{V}$, which forms a family of complex manifolds $\{W_t | t \in M\}$, W_t being a submanifold of V_t and $W_0 = W$?

In the case where W is of co-dimension 1 in V_0, the problem is divided into two parts: extension of the line bundle $[W]$ defined over V_0 to a family of bundles over $\mathcal{V}$, and the extension of the cross section of $[W]$ defining the divisor W to a family of cross sections.

As for the first part, Kodaira and Spencer gave a condition in [3], § 13. We shall give here another condition, which may be called the differentiated form of theirs.

§ 1. $\barwedge$ operation of Fröhlicher and Nijenhuis

In [1] and [2], Fröhlicher and Nijenhuis defined a kind of multiplication between a scalar differential form and a vector differential form, and studied its properties.

Let X be a differentiable manifold and ω, L be scalar and vector differential forms of degrees q and l respectively, then $\omega \barwedge L$ is a scalar form of degree $(q+l-1)$ defined by