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On the nilpotence of the hypergeometric equation

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Introduction

Let T be an arbitrary scheme, S a smooth T-scheme and $\mathcal{M}$ a quasi-coherent $\mathcal{O}_S$-module. A T-connection on $\mathcal{M}$ is by definition a homomorphism of $\mathcal{O}_S$-modules:

$$\nabla: \mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S) \longrightarrow \mathcal{E}nd_{\mathcal{O}_T}(\mathcal{M})$$

which satisfies the "product formula":

$$\nabla(D)(sm) = s\nabla(D)(m) + D(s)m$$

for sections D of $\mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S)$, s of $\mathcal{O}_S$ and m of $\mathcal{M}$ over an open subset $U \subseteq S$. A section m of $\mathcal{M}$ over U is called horizontal if $\nabla(D)(m)=0$ for all D's, derivations on open subsets of U. Both $\mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S)$ and $\mathcal{E}nd_{\mathcal{O}_T}(\mathcal{M})$ are $\mathcal{O}_T$-Lie-algebras via the commutator bracket. The connection is called integrable if it is a Lie-algebra homomorphism. The obstruction to a connection being integrable is the curvature homomorphism $K: \bigwedge^2 \mathcal{D}er_{\mathcal{O}_T}(\mathcal{O}_S, \mathcal{O}_S) \rightarrow \mathcal{E}nd_{\mathcal{O}_S}(\mathcal{M})$ defined by $K(D \wedge D') = [\nabla(D), \nabla(D)] - \nabla([D, D'])$. Henceforth we will deal only with integrable connections.

A horizontal morphism ϕ between modules with connection