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On potential densities of one-dimensional Lévy processes

By

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§1. Introduction

In this paper, we will study some behaviors near the origin of the derivatives of potential densities of some typical one-dimensional Lévy processes.

In the study of one-dimensional Lévy processes, their potential densities play an important role. For example, there is a close relation between the hitting probability for a single point and properties of potential densities: roughly, we can say that the positivity of hitting probability for a single point is equivalent to the existence of a bounded potential density and the regularity of a single point is equivalent to the existence of a bounded continuous density. These facts were well known and used in the study of stable processes (cf. Kac [3]) and have been established for general one-dimensional Lévy processes by Kesten [4] and Bretagnolle [1]. We note that Port and Stone [6] proved independently the existence of continuous densities (and hence, the regularity of a sinlge point) for asymmetric Cauchy processes.

Even in the case when a continuous potential density exists, its derivative behaves quite differently and it is our purpose of the present paper to study the behavior of derivatives near the origin for several one-dimensional Lévy processes. The behavior of derivatives reflects some aspects of the hitting of sample paths to a given point as is explained in Ikeda and Watanabe [2].