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Dependence of local homeomorphisms and local C^r-structures

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Introduction

Let $\Sigma(X)$ be the set of all C^r-structures on a topological manifold X. The study of the diffeomorphism classes of $\Sigma(X)$ has been an important subject in differential topology. We, however, consider $\Sigma(X)$ itself paying attention to its dependence relation ($\subset$) defined below. We give some results which are chiefly reduced to a local theory of homeomorphisms of $\mathbf{R}^n$. We begin by the following problems.

Problem I G. *For given C^r-structures $\mathscr{D}$, $\mathscr{D}' \in \Sigma(X)$, can we find a third $\mathscr{D}'' \in \Sigma(X)$ such that $\mathscr{D} \subset \mathscr{D}''$, $\mathscr{D}' \subset \mathscr{D}''$?*

Problem II G. *For given C^r-structures $\mathscr{D}$, $\mathscr{D}' \in \Sigma(X)$, can we find a third $\mathscr{D}'' \in \Sigma(X)$ such that $\mathscr{D}'' \subset \mathscr{D}$, $\mathscr{D}'' \subset \mathscr{D}'$?*

These problems are quite raw and more suitable presentations will be found according to the stages of our study. First, we localize the problems.

By a *local C^r-structures* on $\mathbf{R}^n$ we mean the germ at 0 of a C^r-structure of a neighbourhood of $0 \in \mathbf{R}^n$ (we shall give a more detailed definition in Section 1). By a *local homeomorphism*(1) of $\mathbf{R}^n$ we mean the germ at 0 of that homeomorphism between neighbourhoods of 0

(1) We use this term following Sternberg, who investigated local homeomorphisms in connection with the theory of flow and found normal forms of conjugate classes of local diffeomorphisms.