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The integral cohomology ring of F_4/T and E_6/T

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§0. Introduction and statement of the result.

Let G be a compact, connected, simple Lie group and T its maximal torus. As is well known [7], G/T has no torsion and its Poincaré poylnomial is

$$P(G/T; t)=\prod_{i=1}^{l}\frac{1-t^{2m_i}}{1-t^2}$$

where $(2m_1-1, \ldots, 2m_l-1)$ indicates the degrees of the primitive elements of $H^*(G; Q)$. Thus the additive structure of $H^*(G/T; Z)$ is known. Furthermore its ring structure is known for $G=U(n), Sp(n)$, G_2 [2], [7] and probably for $G=SO(n)$. The purpose of this paper is to determine the ring structure of $H^*(G/T; Z)$ for $G=F_4$ and E_6, where F_4 and E_6 are simply connected, compact exceptional Lie groups of rank 4 and 6 respectively.

Throughout the paper $H^*(X)$ always denotes the integral cohomology ring of X and

$$\sigma_i(t_1, t_2, \ldots, t_n)$$

is the i-th elementary symmetric function on n variables $t_1, t_2, \ldots, t_n$. Then our main results are stated as follows.

Theorem A.

$$H^*(F_4/T)=Z[t_1, t_2, t_3, t_4, \gamma_1, \gamma_3, w]/(\rho_1, \rho_2, \rho_3, \rho_4, \rho_6, \rho_8, \rho_{12})$$