

# The explosion problem of branching Lévy processes

By

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(Communicated by Prof. Watanabe, May 1, 1975)

## §1. Introduction and Preliminary

In the paper [9] the author discussed the explosion problem of branching stable processes in connection with the problem of uniqueness and non-uniqueness of solutions for a class of non-linear integral equations (the  $S$ -equations of branching stable processes). The present paper is an extension of [9], and is devoted to strengthening of the conditions for explosion. First we shall give two sufficient conditions for explosion for a class of branching Lévy processes (Propositions 1 and 2 of §2). Then we shall apply the conditions to branching stable processes and branching Poisson processes, and explicitly distinguish explosion case from non-explosion case (Theorems 1 and 2 of §3, and Theorem 3 of §4). Finally we shall prove two comparison theorems for explosion of branching Lévy processes, and give some application of them (Theorems 4 and 5 of §5).

1. Let  $X=(W, X_t, P_x, x \in R)$  be a Lévy process on the real line  $R$ , that is, a standard Markov process on  $R$ , homogeneous in space and time and characterized by the representation

$$E(\exp(i\xi X_t)) = \exp\{t\Psi(\xi)\}^{1)},$$

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1) We denote  $P_0(\cdot)$  and  $E_0(\cdot)$  related to a Lévy process by  $P(\cdot)$  and  $E(\cdot)$ , respectively.