J. Math. Kyoto Univ. (JMKYAZ)
16-3 (1976) 639-652

Finsler manifolds modeled on a Minkowski space

By

Yoshihiro ICHIJYŌ

(Communicated by Prof. Toda, Dec. 17, 1975)

As is well known, the tangent space at any point of a Riemannian manifold is a Euclidean space. On the other hand, in Finsler manifolds and modern Banach manifolds, the tangent space at any point is a Minkowski space, but the tangent spaces at two distinct points are, in general, not the same Minkowski space. Hence it seems significant to us to study the manifolds with the property such that the tangent spaces at arbitrary points of them are congruent (isometrically linearly isomorphic) to a single Minkowski space. We will call a Finsler manifold with this property as a Finsler manifold modeled on a Minkowski space.

The main purpose of the present paper is to develop the theory of Finsler manifolds modeled on a Minkowski space and to give some examples of them.

First, we shall introduce a Minkowski norm and linear Lie groups leaving the Minkowski norm invariant. In the section 2, we shall define the notion of $\{V, H\}$-manifolds, where V is a Minkowski space and H is a linear Lie group leaving the Minkowski norm invariant. We shall show that the $\{V, H\}$-manifold offers an example of a Finsler manifold modeled on a Minkowski space. In the section 3, it will be proved that a $\{V, H\}$-manifold is a generalized Berwald space defined by Hashiguchi [7], and also a generalized Berwald space is a Finsler manifold modeled on a Minkowski space. We shall consider, in the section 4, a condition for a Finsler manifold to be a Finsler