Singular hydrodynamical continuations of finite Riemann surfaces\dagger)

Dedicated to Professor Yukio Kusunoki on his sixtieth birthday

By

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Introduction

The present study arose, in close relationships to a series of our investigations [16], [17] and [18], from an attempt to embed an arbitrary open Riemann surface of finite genus into another closed Riemann surface of the same genus, so that the prolongation of the surface accompanies that of some global meromorphic uniformizer of the given surface\dagger).

It is a well known fact that an open Riemann surface of finite genus is prolongable anyway up to some closed Riemann surface of the same genus (see Bochner [2]). It belongs, however, to comparatively recent questions, what variety of like prolongations occur, or whether we are always able to find among them the one endowed with somewhat distinguished features such as extremalities. As for the first problem the readers are referred to, e.g. Mori [10], Heins [5], Grunsky [3], [4], Oikawa [11], and for the second to Ioffe [6], Timmann [21], Shiba [16], [17], Shiba-Shibata [18], and others. They are also intimately connected with the problems of conformal mappings and of realizations of Riemann surfaces, to which significant contributions have been made above all by the mathematicians in Hannover: Tietz [19], [20], Köditz-Timmann [7] and Schmieder [14], [15].

In our preceding papers [16] and [18] we have shown the following: Given an open Riemann surface $R$ of finite genus together with a special kind of single-valued meromorphic function $f$ (called an S-function — a generalization of the classical Strömungsfunktion) on it, there is a closed Riemann surface $R^* \Rightarrow R$ of the same genus as $R$ such that $f$ is continued holomorphically up to $R^* \subset \partial R$. More precisely, for the given $R$ and $f$ there exists a closed Riemann surface $R^*$, a conformal

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\dagger) We are concerned here with no more than the extensions with the property 'non-increase of genus'.