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Mixed problems for pluriparabolic equations

By

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Petrowski considered the well-posedness of Cauchy problems for evolution equations with t-dependent coefficients, and introduced two typical subclasses —strictly hyperbolic and p-parabolic ([1]). Volevich-Gindikin considered Cauchy problems for pluriparabolic equation—the third subclass of evolution equations ([2]). Finally, Volevich proved the well-posedness of Cauchy problems for $\mathcal{H}$-correct evolution equations with (t, x)-dependent coefficients, where the class of $\mathcal{H}$-correct evolution equations is a subclass of evolution equations containing the above three classes ([3]). On the other hand, there are little works on mixed problems for evolution equations other than hyperbolic or parabolic equations ([4], [5]).

In this paper, the author considers the mixed problems for pluriparabolic equations. She uses the energy method, where the main tools are the pseudo-differential operators with weight functions ([6]). She uses two types of weight functions and pays attentions to the separation of two types of symbols. To get the energy inequalities, the choice of energy forms is based on the technique used in [7].

A typical example of pluriparabolic mixed problems is given by

$$\begin{cases} \partial_t u = -\partial_x u + \partial_y^2 u + f & (t>0,\ x>0,\ -\infty<y<+\infty), \\ u|_{x=0} = g & (t>0,\ -\infty<y<+\infty), \\ u|_{t=0} = h & (x>0,\ -\infty<y<+\infty). \end{cases}$$

More general pluriparabolic equations of order 1 with respect to ∂_t are investigated under the name of ultraparabolic equations ([8], [9]).

§1. Pseudo-differential operators with weight functions.

1.1. For $\rho=(\rho_1, \rho_2, \cdots, \rho_n)$ $(\rho_i>0)$, we say that $\lambda(\xi)$ $(\geqq 1)$ is a weight function if

$$|\partial_\xi^\alpha \lambda(\xi)| \leqq C_\alpha \lambda(\xi)^{1-\rho\cdot\alpha}.$$

Moreover, we say that $a(x, \xi) \in S_{\lambda,\rho}^m$ if

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