J. Math. Kyoto Univ. (JMKYAZ)
26-3 (1986) 357–374

Magnetic Schrödinger operators with compact resolvent

By

Akira IWATSUKA

1. Introduction.

In this paper we shall consider the magnetic Schrödinger operators

$$L_V(\boldsymbol{a}) = -\sum_{j=1}^{n}\left(\frac{\partial}{\partial x_j}-ia_j\right)^2+V \tag{1.1}$$

where a_j and V are the operators of multiplication by real-valued functions $a_j(x)$ and $V(x)$, respectively. We assume

$$\begin{cases} a_j(x)\in L^2_{\mathrm{loc}}(\boldsymbol{R}^n) & \text{for } j=1,\cdots,n\,, \\ V(x)\in L^1_{\mathrm{loc}}(\boldsymbol{R}^n) & \text{and } V(x)\geqq 0\,, \end{cases} \tag{1.2}$$

where, for $p\geqq 1$ and an open set Ω in $\boldsymbol{R}^n$, $L^p_{\mathrm{loc}}(\Omega)=\{f\,|\,\zeta f\in L^p(\Omega)$ for all $\zeta\in C_0^\infty(\Omega)\}$, $L^p(\Omega)$ being the space of complex-valued measurable functions f on Ω with $\|f\|_{L^p(\Omega)}=[\int_\Omega |f|^p]^{1/p}<\infty$ and $C_0^\infty(\Omega)$=the space of C^∞ complex-valued functions with compact support in Ω. Consider the form in the Hilbert space $L^2(\boldsymbol{R}^n)$

$$\begin{aligned} h_{\boldsymbol{a},V}(\phi,\psi) &= (L_V(\boldsymbol{a})\,\phi,\psi) \\ &= \sum_{j=1}^{n}(\Pi_j(\boldsymbol{a})\,\phi,\Pi_j(\boldsymbol{a})\,\psi)+(V\phi,\psi) \end{aligned} \tag{1.3}$$

for ϕ, $\psi\in Q(h_{\boldsymbol{a},V})\equiv$" the form domain of $h_{\boldsymbol{a},V}$ "$\equiv C_0^\infty(\boldsymbol{R}^n)$, where $(u,v)=\int_{\boldsymbol{R}^n}u\bar{v}$ and

$$\Pi_j(\boldsymbol{a}) = \frac{1}{i}\frac{\partial}{\partial x_j}-a_j\,.$$

Then it is known (see, e.g., Leinfelder and Simader [5]) that $h_{\boldsymbol{a},V}$ is closable and its form closure $\bar{h}_{\boldsymbol{a},V}$ is a non-negative symmetric form such that:

$$\begin{cases} Q(\bar{h}_{\boldsymbol{a},V}) = \{u\in L^2(\boldsymbol{R}^n)\,|\,\Pi_j(\boldsymbol{a})\,u\in L^2(\boldsymbol{R}^n) \quad \text{for } j=1,\cdots,n \\ \qquad \text{and} \quad V^{1/2}u\in L^2(\boldsymbol{R}^n)\}\,, \\ \bar{h}_{\boldsymbol{a},V}(u,v) = \sum_{j=1}^{n}(\Pi_j(\boldsymbol{a})\,u,\Pi_j(\boldsymbol{a})\,v)+(V^{1/2}u,V^{1/2}v)\,, \end{cases} \tag{1.4}$$

Received February 14, 1985