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Estimates for degenerate Schrödinger operators and hypoellipticity for infinitely degenerate elliptic operators

By

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Introduction and main theorems

In Chapter II of [1] Fefferman and Phong estimated the eigenvalues of Schrödinger operators $-\Delta+V(x)$ on $\boldsymbol{R}^n$ by using the uncertainty principle. Inspired by their idea, in the present paper we give some L^2-estimates for degenerate Schrödinger operators of higher order, which are versions and extensions of Theorem 4 in Chapter II of [1]. As applications, we consider the hypoellipticity of infinitely degenerate elliptic operators of second order. Some parts of the present paper (Theorem 1, 2 and 6 below) are announced in [13].

Consider a symbol of the form

$$a(x, \xi)=\sum_{k=1}^{n} a_k(x)|\xi_k|^{2\mu_k}+V(x), \qquad x\in\boldsymbol{R}^n, \tag{1}$$

where μ_k are positive rational numbers, $V(x)\geqq 0$ belongs to $L_1^{\mathrm{loc}}(\boldsymbol{R}^n)$ and

$$\begin{cases} a_1(x)=1, \\ a_k(x)=\sum_{j=1}^{k-1}|x_j|^{2\kappa(k,j)} \qquad \text{for } k\geqq 2. \end{cases} \tag{2}$$

Here $\kappa(k, j)$ are non-negative rational numbers. If $x_0\in\boldsymbol{R}^n$ and if $\delta=(\delta_1, \cdots, \delta_n)$ for $\delta_j>0$, we denote by $B_\delta(x_0)$ a box

$$\{(x, \xi);\ |x_j-x_{0j}|\leqq\delta_j/2,\ |\xi_j|\leqq\delta_j^{-1}/2\}. \tag{4}$$

Clearly the volume of $B_\delta(x_0)$ is equal to 1. Let C denote a set of boxes $B_\delta(x_0)$ for all x_0 and all δ. We denote by $\boldsymbol{m}_l(\cdot)$ the Lebesgue measure in R^l. We set $m_k=\mu_k-1$ if μ_k is integer and $m_k=[\mu_k]$ otherwise. Set $m_0=\sum_{k=1}^{n} m_k$.

Theorem 1. *Let $a(x, \xi)$ be the above symbol and let $W(x)$ be a real-valued continuous function in $\boldsymbol{R}^n$. Assume that there exists a constant $1-2^{-m_0}<c\leqq 1$ such that for any $B=B_\delta(x_0)\in C$*

$$\boldsymbol{m}_{2n}(\{(x, \xi)\in B\ ;\ a(x, \xi)\geqq\max_{\pi(B^{**})} W(x)\})\geqq c, \tag{4}$$

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