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Mixed problems or Cauchy problems for semi-degenerate hyperbolic equations of 2-nd order with a parameter

By

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Introduction.

Let us consider linear hyperbolic operators of 2-nd order with real coefficients:

$$L=L(t, x; \partial_t, \partial_x)=\partial_t^2-2\sum_{j=1}^{n} a_{j0}(t, x)\partial_j\partial_t-\sum_{j,k=1}^{n} a_{jk}(t, x)\partial_j\partial_k$$

$$+b_0(t, x)\partial_t+\sum_{j=1}^{n} b_j(t, x)\partial_j+c(t, x)$$

in $I\times\Omega=[0, T]\times R_+^n=\{0\leqq t\leqq T,\ x_1>0,\ x'=(x_2, \cdots, x_n)\in R^{n-1}\}$, where $a_{jk}=a_{kj}$ and $\partial_j=\partial/\partial x_j$. It is well known that the mixed problem:

$$\text{(M.P)}\qquad \begin{cases} Lu=f & \text{in } I\times\Omega, \\ u|_{x_1=0}=g_0 & \text{on } I\times\partial\Omega. \\ u|_{t=0}=u_0, \quad \partial_t u|_{t=0}=u_1 & \text{on } \Omega \end{cases}$$

is well posed, if

i) $\displaystyle\inf_{I\times\Omega\times S^{n-1}} \sum_{j,k=1}^{n} a_{jk}(t, x)\xi_j\xi_k>0,$

ii) $a_{jk}, b_j, c\in \mathcal{B}^\infty(I\times\Omega)$

are satisfied. How about the problem if i) and ii) are replaced by

i)′ $\displaystyle\inf_{I\times\Omega_\varepsilon\times S^{n-1}} \sum_{j,k=1}^{n} a_{jk}(t, x)\xi_j\xi_k>0 \qquad (\text{any } \varepsilon>0),$

ii)′ $a_{jk}, b_j, c\in \mathcal{B}^\infty(I\times\Omega_\varepsilon) \qquad (\text{any } \varepsilon>0),$

where $\Omega_\varepsilon=\Omega\cap\{x_1>\varepsilon\}$? In this paper, assuming i)′ and ii)′, we consider two cases. One is a degenerate case, when i) is not satisfied, and the other is a singular case, when ii) is not satisfied. Their typical examples are as follows:

(I) $L=\partial_t^2-\rho\partial_1^2-\partial_2^2-(\mu+1)\partial_1,$

(II) $L=\partial_t^2-\partial_1^2-\partial_2^2-(\mu+1)\rho^{-1}\partial_1,$

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